The Role of Integrative Technologies as a "Force Exponent" on Military Capability

(Preliminary draft)

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Abstract

Integrative technologies are information and communication technologies that enable separate individual systems to work in a joint, coordinated, and synergistic fashion as a single holistic system. An increasingly growing share of defense R&D expenditures are devoted to the development and fielding of integrative technologies, such as Command and Control systems, communications systems, etc. This study explores the optimal defense budget allocation to procurement and R&D of weapon systems and to the development of integrative technologies. We develop an optimization framework that captures the main characteristics of the problem. We then use it to derive the optimal allocation, and to analyze it. To demonstrate the viability of the model, we apply it to the US military.

1. Introduction

Since the end of World War II, many countries spend a considerable share of their defense budgets on Research and Development activities (R&D). These expenditures often represent a significant share of the entire government funded R&D in those countries. For example, the United States allocated almost 15% of its $400 Billion defense budget to defense R&D in 2003 (DOD, 2003), representing more than 50% of the federally funded R&D activities (AAAS, 2002).

The need to appropriately allocate budgets to defense R&D has long been recognized (e.g., Hitch and McKean, 1963). Nevertheless, the literature on economics of defense R&D is scarce, and offers but a few models and approaches to meet this need (see, for example, Stoneman, 1987; Rogerson, 1990; Hirao, 1994; Sandler and Hartley, 1995; Lichtenberg, 1995; Garcia-Alonso, 1999; Gordon, 2003). Moreover, The existing models all share a common view of the purpose of defense R&D: military forces consist of various weapon systems, *each* characterized by its quantity and quality; defense R&D enhances the quality of these systems. For example, the quality of a fighter plane is improved by equipping it with better missiles, better avionics, or by replacing it with an advanced fighter plane altogether. The higher is the quality of a fighter plane, the more effective missions it is capable to perform, thus increasing overall military capability.

This view, however, fails to acknowledge the rapidly increasing role of *integrative technologies* in modern military forces. Integrative technologies are information and communication technologies that enable *separate individual systems* to work in a joint, coordinated, and synergistic fashion as a *single holistic system*. Common examples of integrative technologies in the military are Command and Control (C^2) systems, global communications networks, cross-service information systems, etc.

The value of these technologies stems not only from the network they create, which is attainable using other simple means (e.g., tactical radios and telephones), but also from the *quality of integration* they provide. The higher is the quality of integration, the more effective is the overall operation of the networked units. Analysis of US expenditure data, presented in this study, shows that roughly 20% of defense R&D expenditures in the last few years are devoted to integrative technologies.

Integrative technologies exhibit three key characteristics: first, integrative technologies have a "*military-wide*" effect; increased quality of integration benefits the entire military, and not any particular component of it. Second, integrative technologies may be able to exhibit an exponential "*network*" effect; they influence the quality of the interactions between systems, and the number of connections is exponential in the number of systems (size of the network). Finally, being a relatively new phenomenon, the development of integrative technologies may still exhibit increasing returns. Since integrative technologies will eventually exhibit decreasing returns, like other maturing technologies, their overall *returns profile* should be modeled as an *S-shape (convex-concave)* (Christensen, 1992).

This paper aims to explore the optimal budget allocation to defense R&D, and particularly to the development of integrative technologies. In order to meet this objective, the paper develops a static model (optimization framework), in which defense budget is allocated between R&D and procurement in order to maximize military capability. The model is then applied to the US Defense budget. The analysis of the US budget, based on a unique data set that was developed in this study from publicly available US defense budget data, uses non-linear regression techniques to calibrate the model's parameters and to test its central assumptions.

The primary contribution of the study is in providing a first comprehensive analysis of the economics of integrative technologies[1]. The study develops and successfully applies an analytical framework that captures the defining characteristics of integrative technologies, and provides a simple budget allocation procedure with a unique optimal solution.

The paper proceeds as follows. Section 2 critically reviews the existing literature on defense budget allocation, defense R&D and, particularly, integrative technologies. Section 3 presents a baseline model of defense budget allocation to R&D and procurement. Section 4 then extends this model to include integrative technologies. Section 5 analyzes the characteristics of the optimal solution, and Section 6 presents the application to the US military case. Section 7 concludes and provides suggestion for further research.

2. Background, Definitions and Literature Review

Most 'rational' models of defense[2] assume that governments maximize social welfare. Social welfare is, generally, a function of national security and civilian outputs (Smith, 1980). Since national security is unobservable econometrically (Smith, 1995), it is typically modeled, in the economic literature, as a function of the military

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[1] The economic model of integrative technologies is not limited to the defense sector. It can be applied to any hierarchical organization that employs information systems technology in its routine operation. Common examples to such technologies are email, videoconferencing, Intranet, Knowledge Management systems and Enterprise Resource Planning (ERP) systems. These technologies play an increasingly important role in organizations.

[2] Alternatively, 'bureaucratic' models focus on the conflicting interests of various government agencies (Olvey, Golden and Kelly, 1984).

capabilities of a nation and its rivals[3]. Common examples are the difference between the military capabilities (e.g., Levine and Smith, 1997; Garcia-Alonso, 1999), and the ratio between the military capabilities (e.g., Golde and Tishler, 2002; Mantin and Tishler, 2005).

There are many factors determining military capability, such as quantity and quality of weapon systems and personnel, level of command, level of training, level of maintenance, force morale, and so forth (Handel, 1985). These can be used as inputs to a "military capability" function, which relates them to a "subjective figure of merit"[4] (Hildebrandt, 1999). Intriligator (1975) uses the quantities of nuclear warheads as a measure of the military capability of the US and the Soviet Union in his arms race. Similarly, Levine, Sen and Smith (1994) use the quantity of a homogenous defense product in their model of the arms trade market. They comment that this could be thought of as a quality corrected aggregate of several weapon systems, but do not discuss how such an aggregate is formed.

Production function formulations are often used to aggregate quantities and qualities of various weapon systems into a single measure of military capability[5]. Hirao (1994) defines the military capability function as a Cobb-Douglas production function, whose arguments are quantity and quality. This formulation assumes, however, the existence of a single type of weapon system. Garcia-Alonso (1999) proposes a CES formulation (Varian, 2002) in which various weapon systems are imperfect substitutes, and the

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[3] The military capability of a nation's allies is also an important factor of national security allies (Gates and Terasawa, 2003). This paper does not address the issue of allies.

[4] Hildebrandt (1999) refers to this function as a "military utility function", and contrasts it with a "military production function" whose output is some objectively measurable indicator of military output.

[5] See Olvey, Golden and Kelly (1984) for a discussion on the plausibility of using production function in this context.

capability of each weapon system is a direct multiplication of its quantity and quality. Dunne, Garcia-Alonso, Levine and Smith (2002) extend this model by further dividing military goods: there are types of weapon systems that are poor substitutes (e.g. tanks and planes). Within each type, there are a variety of weapon systems, which are relatively close substitutes (e.g. various fighter planes). Their model uses a nested CES formulation, in which military capability is a function of the capability of the various types. The capability of each type is in turn a CES function of the variety of weapon systems in this type. The elasticity of substitution for each function is chosen to reflect the assumptions described above. Although this model provides an aggregate military capability that takes into account the variety of weapon systems, it does not account for the effect of integrative technologies.

Gordon (2003) measures the capability of several Middle Eastern air forces using a nested weighted-average approach. He divides air-force capability to offensive and defensive capabilities; each is then divided to categories (such as weapon systems, manpower, infrastructure and "system of systems"[6]). Each category is further divided to sub-categories and then to groups. In each stage, the various factors are weighted using expert opinions, and scored using data, where applicable, and expert opinions. Although this approach accounts for the various factors determining military capability, its formulation is highly limiting from a resource allocation perspective; the assumption of constant weights implies that each factor has a constant marginal contribution to military capability. Consequently, the optimal behavior is to concentrate all defense expenditure in the factor that provides the maximal marginal contribution per unit price, which is, of course, unrealistic. In addition, the heavy

[6] The term "system of systems" is frequently used to describe the use of integrative technologies.

reliance of the model on subjective expert opinions limits the validity of the model's results.

In all of the models described above, the perception of the relation of quality to military capability shares two implicit assumptions: a military consists of distinct weapon systems, the quality of each may be improved[7]; and returns to quantity and quality are decreasing (or non-increasing) [8]. The first assumption disregards the role of *integrative technologies*. Not only is their effect on military capability different in nature than other technologies (as elaborated shortly), but also their extensive use is a new phenomenon, thus it may still enjoy increasing returns.

Integrative technologies are information and communications technologies that enable *separate individual systems* to work in a joint, coordinated, and synergistic fashion as a *single holistic system*. The basic idea of integrative technologies is familiar to any modern organization: phones and inter-office memos, which have been prevalent for decades, allow all the people within the organization to communicate information to each other. That is the basic integration level. Recently, many technologies evolved that improve the *quality of integration*, and change its characteristics. Common examples to such technologies are email, videoconferencing, Intranet, Knowledge Management systems and Enterprise Resource Planning (ERP) systems. These technologies play an increasingly important role in organizations.

Integrative technologies gain a crucial role in modern defense organizations as well. The US military, a global technological leader, puts integrative technologies in a very

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[7] For example, the quality of a fighter plane may be improved by equipping it with better missiles, a better radar, or by replacing it with an advanced fighter plane altogether.

[8] There is actually a third assumption; military capability depends on quantity and quality alone. As mentioned at the beginning of the section, there are other factors, such as level of training, force morale, and so forth; the present analysis also abstracts away from those factors.

high priority (Etter, 2002), and develops concepts that place such technologies at their core (e.g. "Network Centric Warfare", see Alberts, Garstka & Stein, 1999). This is emphasized by formal US Department of Defense (DoD) documents. For example, The Quadrennial Defense Review Report (DoD, 2001), submitted to the US Congress by the Secretary of Defense, specifically requires future forces to be "networked to maximize their combined effects". In addition, it asserts that Information technologies are a "key foundation for the effort to transform US armed forces for the 21st century". From a budget allocation perspective it concludes that future funding will focus on achieving an integrated joint "end-to-end Command, Control, Communication, Computers, Intelligence, Surveillance, and Reconnaissance capabilities".

The application of these concepts is based on a multitude of programs and technologies. For example, the Ground Command and Control System (GCCS) is an operational system, used by the US military, to create a common operational picture of the battlefield across several command echelons (IISS, 2000). The US defense R&D budget includes some 250 programs of integrative systems in the last decade (USD-C, 2003b). Setter (2004) estimates that the US military currently spends approximately 20% of its R&D budget on integrative technologies. Other Western European countries also develop and field integrative systems, such as the UK's Joint Operational Command System (IISS, 2000). Barzilay (2003) reports that the IDF recently decided to develop a central Command and Control system to its ground forces, which is expected to transform the ground forces within the next 5-6 years. This system will then be integrated with the Command and Control systems of the Air Force and the Navy.

There are several factors determining the quality of an integrative system. Alberts and Hayes (2001) characterize integrative systems[9] using three major dimensions: richness, reach and quality of interaction. The *richness* of a system is comprised of its information completeness, correctness, currency, accuracy and consistency. The *reach* of a system is defined by it's geographic range, continuity over time, number of network nodes, sharing of information across levels and across functional areas. The *quality of interaction* depends on whether the system allows for data exchange, voice exchange, text exchange, static images, and video images, as well as on its interactivity and reliability. This list of characteristics measures the quality of a single integrative system. A modern military typically operates many such systems, of varying qualities. Taken together, these systems provide an *aggregate* quality of integration.

Evaluation of integrative technologies, as of information technologies in general, is a difficult task (Shapiro and Varian, 1998; Willcocks and Lester, 1999; Schwartz and Zozaya-Gorostiza, 2000). Since they don't have a direct output, but rather they improve the total output provided by other systems or functions, there is no direct way to measure their contribution.

Gordon (2003) explicitly includes integrative technologies, or "system of systems", as one of the factors in his weighted-average valuation model of military capability, described above. While greatly simplifying the measurement problem, this approach is misaligned with Gordon's own description of integrative technologies not as a separate factor, but rather as synergetic, attaining their value from enhancing the effectiveness of other factors. Sundarraj and Talluri (2003) propose a method for

[9] They specifically refer to "command and control" systems, but their taxonomy fits integrative systems in general.

evaluating componentized ERP systems. These systems include many optional components, which are easily integrated with each other. Although they acknowledge the importance of this integration, they only account for integration costs, and disregard its benefits. Finally, there is a vast body of literature, starting from the seminal paper of Katz and Shapiro (1985), on the value of networks stemming from network externalities. In these models, the value of a network stems from the number of users who are in the network. A common example is that of a fax machine; the more people have faxes, the higher is the value of each fax, because the number of potential senders and recipients increases. Hence, the focus of the network externalities literature is on the *size* of the network, and not on its *quality*.

In order to develop a model of integrative technologies, the difference between improved integration quality and improved "weapon-specific" qualities needs to be addressed. Intuitively, a better fighter plane, for example, can do more effective missions than another, less advanced, fighter. Integrative systems don't achieve operational goals by themselves - but they improve the effectiveness of the systems they integrate. Hence, integrative technologies have a "*military-wide*" effect[10]. Moreover, because the number of interconnections between the other systems in the network is exponential in the number of systems, better integration contributes exponentially. This "*network*" effect causes integrative to be synergetic, i.e. the whole is greater than the sum of its parts.

As noted earlier, the extensive use of integrative technologies is a recent phenomenon, based on lessons-learned from the 1991 Gulf war (Ben Israel, 2003). The marginal value of such new technologies is often increasing (Loch and Kavadias, 1992); the

[10] A "military-wide" effect is also shared by other technological infrastructures. It is the combination with the "network" effect that distinguishes integrative technologies from other technologies.

initial quality level of the technology provides small operational benefits, as the technology is not yet stable, and its best uses are not yet understood. Once it overcomes these "childhood" effects, it's operational benefits are quickly realized, and implemented. Only when the technology matures it exhibits the standard decreasing marginal value, after the main operational "breakthroughs" were achieved, and gradual improvements provide ever-decreasing benefits. Thus, through its entire life cycle, the value of a technology is *S-shaped*: convex at first, and then concave, converging asymptotically to some physical limit (Christensen, 1992).

Analysis of optimal expenditures on defense R&D in general, and on R&D of integrative technologies in particular, requires a suitable analytic framework[11]. Stoneman (1987) proposes a two-period model in which the government maximizes social welfare, a function of private consumption and security. Security itself is a function of military expenditure and the strategic environment. R&D that is performed in the first time period enhances security in the second period. Social welfare is then maximized subject to a linear national budget constraint for each time period. The model yields the intuitive result that the share of R&D in military expenditure is positively related to the effectiveness of the R&D efforts, and negatively related to the effectiveness of alternative military expenditures. The attempt to address the R&D budget allocation problem from a wide national perspective is criticized by Hitch and McKean (1963). They describe the task of conceiving a social welfare function as "impractical and sterile", and recommend tackling the optimization problem of military capability itself (under a given defense

[11] Alternative frameworks are possible. Haapalina (2003) suggests a defense budget allocation scheme that is based on a Decision Support System, incorporating methods of multiple criteria decision-making.

budget), which they find more manageable. Furthermore, although the model does capture some of the main characteristics of defense R&D, it does not address explicitly the effect of R&D on military effectiveness, as well as other defining characteristics of it, such as uncertainty.

In order to circumvent the problem of defining a social welfare function, which is not at the focus of this study, the model presented here assumes that the *defense budget is set exogenously*, thus we are left with the task of maximizing national security. Furthermore, since most models of national security rely on the difference or ratio of the military capability of rivals, an exogenous defense budget implies that a country[12] needs only to *maximize its own military capability under the budget constraint*.

[12] In reality, a "country" is not a unified body, with fully aligned interests and fully informed decision makers. Conflicts of interests and informational asymmetries are present and affect such decisions. Rogerson (1990), for example, demonstrates that the different view of the US Congress and the US military may lead to excessive expenditure on weapon systems' quality at the expense of their quantity. Lipow and Feinerman (2001) arrive at a similar conclusion, but from different reasons. In their model, military officers over-spend on quality to signal their competence, thus increasing their attractiveness to future civilian employers.

3. Baseline Model

The military is given a defense acquisition budget[13] B, which it allocates between weapons procurement and R&D, in order to maximize security over the next planning period (often on the order of 10-20 years). The model assumes the defense budget is determined exogenously, either by maximizing a social welfare function, or by some political or bureaucratic procedure. The model also abstracts away from agency biases, and the military is assumed to be faithfully pursuing its security-maximizing goal.

There are N types of (possibly aggregate) weapon systems. The military decision maker has to decide upon the quantities $x_1,\ldots,x_N$ of each weapon system, and their respective quality levels $q_1,\ldots,q_N$. The total expenditure on all systems must not exceed the exogenously allocated budget level B. The national security of a country is a function of its military's capability and its rivals' military capability. However, assuming the military capability is a scalar measure, and since the acquisition budget is known, strategic considerations are disregarded, and maximum security is achieved by maximizing the country's military capability, denoted M_0. The quantities of weapons and their quality are assumed to fully describe the capability of the military. The general optimization problem is summarized as follows:

$$\begin{aligned} &max\ M_0 = f\left(x_1,\ldots,x_N;q_1,\ldots,q_N\right) \\ &s.t.\ \begin{cases} e\left(x_1,\ldots,x_N;q_1,\ldots,q_N\right) \le B \\ x_1,\ldots,x_N,q_1,\ldots,q_N \ge 0 \end{cases} \end{aligned} \tag{1}$$

[13] The Defense acquisition budget is assumed to be net of operations, maintenance and personnel costs. Alternatively, under the assumption that there are fixed proportions between the cost of weapon systems and the cost of their operations, maintenance and personnel, this cost could incurred in the unit cost of weapons, and the budget then becomes the overall budget.

where $e(x_1,...,x_N;q_1,...,q_N)$ denotes the expenditure on acquisitions. In order to characterize the military capability function and the expenditure function, some further assumptions are required. First, it is assumed that the capability function depends on the capability functions of the various weapon system; hence, the quantity or quality of one weapon system does not directly affect the quantity or quality of another weapon system. Formally,

$$M_0 = f\left(f_1\left(x_1,q_1\right),\ldots,f_N\left(x_N,q_N\right)\right) \tag{2}$$

Following Hirao (1994) we take the capability function of a single system to be a Cobb-Douglas function of quantity and quality:

$$f_i\left(x_i,q_i\right) \equiv x_i^{\alpha_i} q_i^{\beta_i} \tag{3}$$

Where $0 < \alpha < 1$, $0 < \beta < 1$, $\alpha + \beta < 1$, are the elasticities of x_i and q_i respectively.

As for the functional form of the military capability function, M_0, Dunne et al. (2002) use a CES formulation for the case of a variety of weapon systems. This is further simplified in our baseline model, and an additive form is assumed, so the total capability of a military force is the sum of capabilities of its different systems. This formulation ignores any possible synergies between the various systems. These synergies require the use of integrative technologies that are discussed in the next section.

Finally, the model assumes constant prices and a linear expenditure function. All this is summarized in the following program:

$$\max M_0 = \sum_{i=1}^{N} \eta_i x_i^{\alpha_i} q_i^{\beta_i}$$
$$s.t. \begin{cases} \sum_{i=1}^{N} \left(c_i x_i + r_i q_i \right) \le B \\ \forall i \;\; x_i, q_i \ge 0 \end{cases} \tag{4}$$

Where the constants η_i are factors that normalize the capabilities of the various systems to the capability of the first system.

A unique solution to program (4) exists (because of the concavity of the objective function), but it can only be found numerically, and has no explicit analytical form. The following assumption is required to obtain an analytical solution to the program[14]:

Assumption 1: $\rho \equiv \alpha_1 + \beta_1 = \alpha_2 + \beta_2 = \cdots = \alpha_N + \beta_N$ (5)

Assumption 1 implies that the marginal returns on all weapon systems are equal, although the marginal returns on procurement *or* R&D may be different. When (aggregate) systems are mature enough, this is not an implausible assumption, e.g. marginal rate of return when spending on Army systems or Air Force systems may be expected to be similar in present days. This was probably not the case 50 years ago, when aviation technologies were developed much faster than those of ground or naval forces.

Taking Assumption 1 into account yields the following optimal budget allocation scheme:

Proposition 1: Program (4) may be solved in two stages:

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[14] Setter (2004) shows that the intuition is preserved when Assumption 1 does not hold.

1. The optimal budget allocated to each type of weapon system, $\hat{B}_i$, is:

$$\hat{B}_i = \frac{\left(\eta_i D_i^{\rho}\right)^{\frac{1}{1-\rho}}}{\sum_{i=1}^{N}\left(\eta_i D_i^{\rho}\right)^{\frac{1}{1-\rho}}} \cdot B \tag{6}$$

Where $$D_i \equiv \left(\frac{\frac{\alpha_i}{\rho}}{c_i}\right)^{\frac{\alpha_i}{\rho}} \left(\frac{\frac{\beta_i}{\rho}}{r_i}\right)^{\frac{\beta_i}{\rho}}, \rho \equiv \alpha_i + \beta_i \tag{7}$$

2. For each system, the optimal allocation of the budget $\hat{B}_i$ between x_i and q_i is:

$$\begin{aligned} \hat{x}_i &= \frac{\alpha_i}{\rho}\frac{\hat{B}_i}{c_i} \\ \hat{q}_i &= \frac{\beta_i}{\rho}\frac{\hat{B}_i}{r_i} \end{aligned} \tag{8}$$

Proof: See Appendix.

Intuitively, the use of a two-stage scheme is permitted due to the assumption that overall capability is a function of the capabilities of the various systems. Given that, a backwards induction method may be used: At the second stage, a certain budget is allocated to procurement and R&D of each weapon system, a standard Cobb-Douglas solution. Using this result, the indirect capability function of each system is calculated, and the total budget is allocated between the various systems in a manner that guarantees equal marginal benefits from all systems.

Corollary 1.1:

$$\hat{M}_0 = \hat{f}(B) = (D \cdot B)^{\rho} \tag{9}$$

Where:

$$D = \frac{\left[\sum_i \left(\eta_i D_i\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{1}{\rho}}}{\sum_i \left(\eta_i D_i^{\rho}\right)^{\frac{1}{1-\rho}}} \tag{10}$$

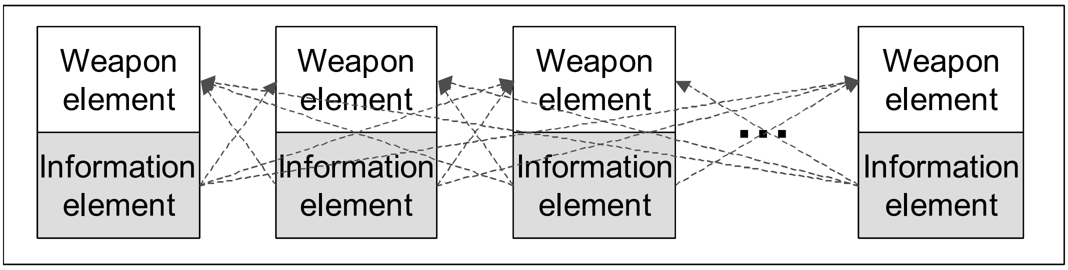
Proof: See Appendix.

The indirect military capability function specifies the military capability obtained for the optimal allocation of a given budget.

4. A model of Integrative Technologies

Integrative technologies allow modern armed forces to operate in a joint, integrated and coordinated fashion through the use of state-of-the-art technologies of command and control, intelligence, targeting, information systems, and so forth. In order to model the impact of integrative technologies, their operational role needs to be clarified first. Figure 1 presents a schematic model whose building blocks are weapon systems, or operational units. Each system (or unit) consists of a "weapon element", which affects the rival force, and an "information-processing element", which gathers, processes and disseminates relevant information to the respective weapon element. Principally, integrative technologies interconnect every "information-processing" element with any other "weapon element". When the quality of integration is high, accurate and relevant information reaches each weapon element on time, and in the right format.

Figure 1: A schematic model of integration



As the figure schematically illustrates, integrative technologies have two distinct characteristics that together distinguish them from other technologies: First, the operational contribution of integrative technologies is *military-wide*, unlike that of "weapon-specific" technologies. Second, they have an *exponential network-effect* on the military capability, stemming from their ability to interconnect **all** the other systems. In addition, integrative technologies are currently young, and may therefore still enjoy *increasing returns*.

In order to study the effect of integrative technologies on budget allocation decision, the extended model is required to exhibit the aforementioned characteristics. The model is an extension of the baseline model, and as such retains all the assumptions of the baseline model, unless explicitly stated otherwise. In particular, a military decision maker has to allocate an exogenously specified defense budget B to develop and procure various weapon systems. In the extended model, the decision maker may also allocate part of the budget to the development and implementation of integrative technologies. The model assumes that the quality of integrative technologies is completely described by a scalar measure, denoted Q_I.

In general, the following program summarizes the model:

$$\max\ M = M\left(x_1,\ldots,x_N;q_1,\ldots,q_N;Q_I\right)$$

$$s.t.\begin{cases} r_I Q_I + \sum_{i=1}^{N}(c_i x_i + r_i q_i) \le B \\ x_1,\ldots,x_N,q_1,\ldots,q_N,Q_I \ge 0 \end{cases} \tag{11}$$

Where M is the extended military capability function, the expenditure function is again assumed linear, and r_I is the constant unit price of integrative technologies.

This model requires some further specification in order to address the distinct characteristics of integrative technologies, as described in the following paragraphs.

Military-wide effect: Implementation of integrative technologies affects the entire military force. It is not specific to any single system or element. From a modeling perspective, the following assumption captures this characteristic:

Assumption 2: The military capability function is separable in Q_I, i.e. the military capability function may be re-written as:

$$M \equiv M\left(f\left(x_1,\ldots,x_N;q_1,\ldots,q_N\right),g\left(Q_I\right)\right) \tag{12}$$

Where f is the military capability achieved from the various weapon systems, as defined in the baseline model, and g governs the contribution achieved from various quality levels of integrative technologies. It is reasonable to require $g \ge 0,\ g' \ge 0$.

Network effect: The second main characteristic of integrative technologies is their being a "force exponent" (as opposed to "force multipliers"), due to their exponential network-effect. The following assumption captures this characteristic:

Assumption 3: The military capability function has the following dependence on $g\left(Q_I\right)$:

$$M = f\left(x_1,\ldots,x_N;q_1,\ldots,q_N\right)^{1+g(Q_I)} \tag{13}$$

This assumption exhibits several other aspects of integrative technologies, besides that of the "force exponent": Since the exponent is defined as[15]: $1+g\left(Q_I\right)$, integrative technologies are not essential to the operation of the military force. The use of pre-planning and standard equipment, such as tactical radios, enables military forces to operate, and integrative technologies only enhance (perhaps significantly) their capability and effectiveness.

In contrast, if the entire budget is allocated to integrative technologies, so that no other weapon systems are acquired, the value of military capability is zero, which is very intuitive: if there is nothing to integrate, the use of integrative technologies in itself is worthless.

S-shaped contribution profile: The contribution of integrative technologies (i.e. the benefit achieved from reaching a higher level of Q_I) is governed by the function *g*. The choice of a specific functional form of *g* depends on the expected development profile of integrative technologies.

Integrative technologies, as a whole, are relatively young compared to other mature military technologies (such as the technology of aviation, radar, missiles, etc.). As such, assuming a concave operational contribution profile is too restricting. On the other hand, convexity of the improvement profile cannot be maintained indefinitely. Christensen (1992) suggested that young technologies evolve in an S-shaped fashion: At first, infrastructure is built, and the benefit from the technology is small. Later on, this benefit rises rapidly, until a point is reached where the contribution profile becomes concave and may converge asymptotically to some physical limit.

[15] This assumption may be generalized by using some parameter instead of '1', but this only complicates notation while not providing any additional insight.

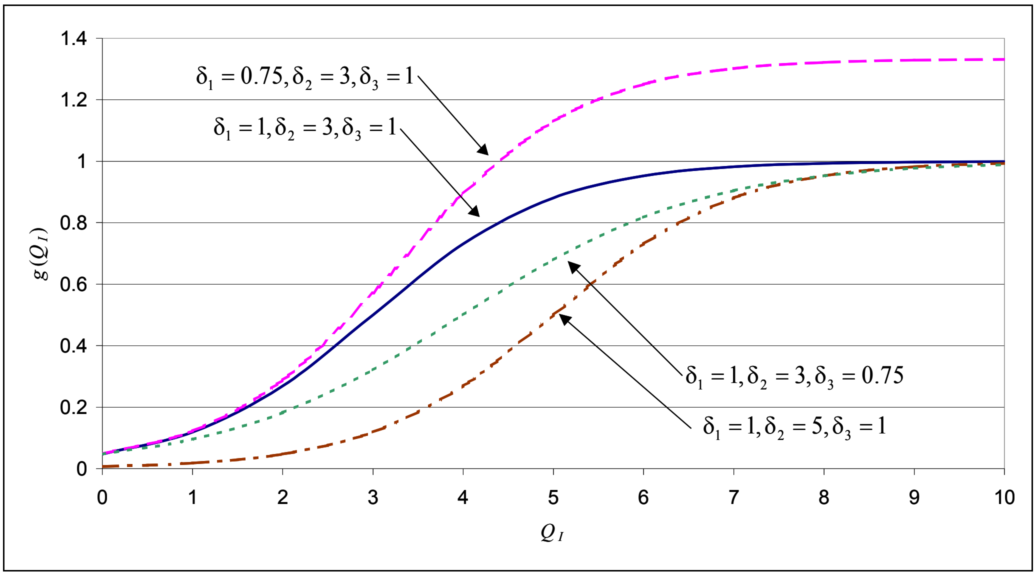
In addition, every military has some initial level of integration (achieved through simple means, such as tactical radios). Finally, mathematical convenience should also play a role in the choice of the functional form. Taking all these considerations into account, the following assumption is taken:

Assumption 4: *g* has the form of the logistic function[16]:

$$g(Q_I) = \frac{1}{\delta_1 + \exp(\delta_2 - \delta_3 Q_I)}; \qquad \delta_i \geq 0 \tag{14}$$

Figure 2 illustrates this function for different values of the three parameters δ_1, δ_2 and δ_3. The S-shape of this function is evident from the figure.

Figure 2: Illustration of the S-shape military contribution function



[16] There are alternative formulations of the logistic functions, but the major results of the dissertation hold for any formulation.

δ_1 is a scale parameter, reflecting the potential contribution of integrative technologies to the military capability. Taking the expression to the limit yields:

$$\lim_{Q_I \to \infty} g(Q_I) = \frac{1}{\delta_1} \tag{15}$$

Thus, the **higher** is δ_1, the **lower** is the potential contribution of integrative technologies. In the remainder of the analysis this parameter is normalized to 1 (extending the analysis to other values is straightforward, and does not change the nature of the results).

δ_2 reflects the effect of the initial level of integrative technologies on military capability, so that the **lower** is δ_2, the **higher** is the initial effect. When the initial level of integrative technologies is higher, the effort required in order to achieve significant contributions from integrative technologies is smaller (seen in the chart as the start of the rapid ascent), representing the know-how obtained by prior acquaintance with these technologies.

Finally, δ_3 is a measurement-units parameter, controlling the slope of the function's ascent with respect to Q_I. The higher is δ_3, the faster the technology approaches its asymptotic value.

As required, the function exhibits an S shape, i.e. it is convex up to some inflection point, after which it is concave, and asymptotically converges to its maximum value. The location of the inflection point, denoted $\tilde{Q}_I$, is given by the following expression[17]:

[17] In order to find the inflection point, we equate the second derivative of g to zero.

$$\tilde{Q}_I = \frac{\delta_2 - \ln(\delta_1)}{\delta_3}. \tag{16}$$

The final optimization problem is therefore:

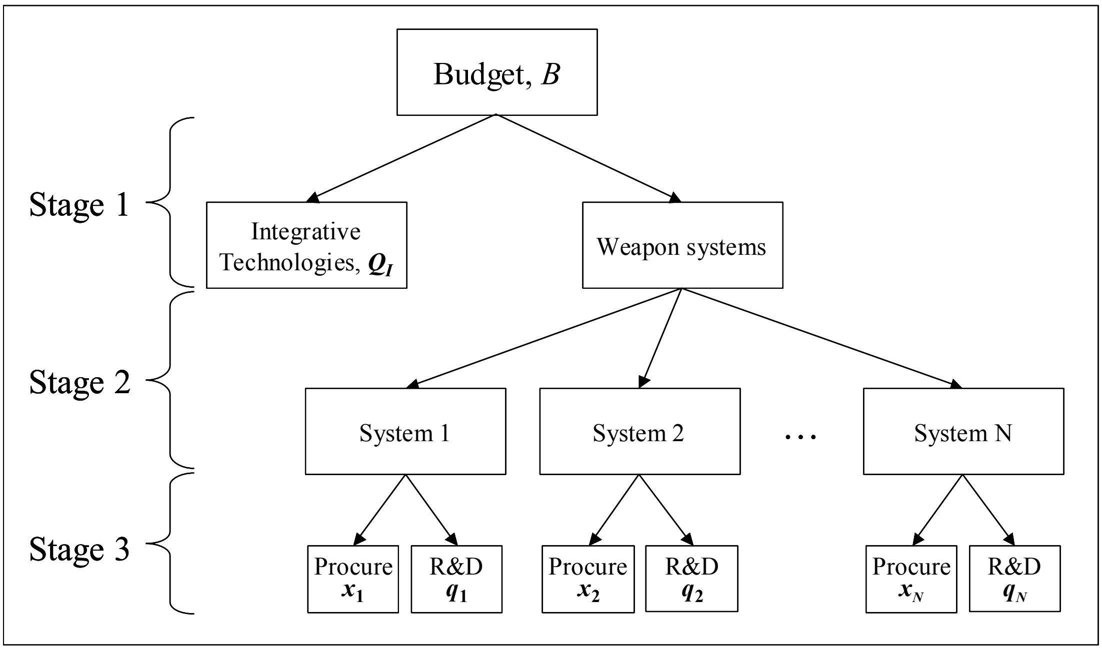
$$\max \ M = f\left(x_1,\ldots,x_N;q_1,\ldots,q_N\right)^{1+g(Q_I)} = \left(\sum_{i=1}^{N} \eta_i x_i^{\alpha_i} q_i^{\beta_i}\right)^{1+\frac{1}{1+\exp(\delta_2-\delta_3 Q_I)}}$$
$$s.t.\begin{cases} r_I Q_I + \sum_{i=1}^{N}(c_i x_i + r_i q_i) \le B \\ x_1,\ldots,x_N,q_1,\ldots,q_N,Q_I \ge 0 \end{cases} \tag{17}$$

Intuitively, the key to the solution procedure is Assumption 2, regarding the separability of the military capability function. Because of this assumption, the chosen level of integrative technologies does not affect the relative shares allocated to procurement and development of the various weapon system. For example, if the optimal expenditure on system 1 is twice as the optimal expenditure on system 2 for a given level of Q_I, this proportion will not change for other levels of Q_I. The effect of Q_I on the optimal values of other variables is indirect: the higher is Q_I, the smaller is the remaining budget, and the smaller are the absolute optimal values of the other variables.

This observation allows using a 3-stage solution procedure, as described in Figure 3. In stage 1, the budget is allocated to "weapon systems" and to "integrative technologies". In stage 2, the "weapon systems" budget is further allocated to the various systems, whereas the allocation to R&D and procurement is done in stage 3.

Furthermore, a closer look at the sub-problem starting at stage 2 reveals that it is **exactly** the problem described by the baseline model, whose solution was already established in the previous section.

Figure 3: A scheme of the 3 stages of the budget allocation procedure



Using the indirect capability function obtained for the baseline model, which relates a given "weapon systems" budget level to the attained military capability, it is now possible to reformulate the first stage of the allocation problem: The military capability is a function of Q_I, the quality of integrative technologies, and of the capability obtained with weapon systems using the remaining budget, which is in itself just a function of Q_I, as seen in Expression (18):

$$\begin{aligned} &\max M = \left(\hat{f}\left(B - r_I Q_I\right)\right)^{1+g(Q_I)} \\ &s.t.\ r_I Q_I \leq B \end{aligned} \tag{18}$$

Where $\hat{f}(\cdot)$ is the indirect capability function, as defined in (9).

The reformulated optimization problem is therefore one of a single variable - Q_I, which greatly simplifies it. Once the optimal Q_I is found, it is straightforward to calculate the remaining budget, and to allocate it according to the solution of the baseline model.

This solution procedure can now be applied to the budget allocation problem (17), taking into account the specific functional forms of *f* and *g*. Using Equation (18), Program (17) can be expressed as:

$$\begin{aligned} \max\ & \hat{M}\left(Q_I\right) = \left[\left(\bar{Q}_I - Q_I\right) r_I D\right]^{\rho\left[1+\frac{1}{\delta_1+\exp(\delta_2-\delta_3 Q_I)}\right]} \\ s.t.\ & Q_I \in [0, \bar{Q}_I] \end{aligned} \tag{19}$$

Where $\hat{M}$ replaces the original objective function *M*. *D* is defined in the baseline model (see Equation 10), and $\bar{Q}_I \equiv B/r_I$ denotes the maximal level of Q_I, when the entire budget is allocated to it.

Note that changes in Q_I have two concurrent effects on $\hat{M}$ in Equation (19), an "operational benefit" and an "operational cost"[18]: The operational benefit stems from the fact that when Q_I increases, the value of the exponential term increases, which increases the value of the objective function. The operational cost of an increase in Q_I is the ensuing decrease of the bracketed term (which represents the decrease of optimal procurement and R&D expenditures on other weapon systems), thus reducing the value of the objective function.

Since *M* is a continuous function in the closed interval $\left[0, \bar{Q}_I\right]$, there is at least one point, denoted $\hat{Q}_I$, in the interval, where it reaches a maximum (Salas and Hille, 1982). This point can be either an internal maximum or a corner one (i.e. at $Q_I = 0$ or $Q_I = \bar{Q}_I$). The corner solutions are examined first:

If $Q_I = 0$, i.e. there is no expenditure on integrative R&D, then:

[18] The benefit and cost are measured in terms (units) of (the natural logarithm of) "military capability".

$$\hat{M}_0 = \hat{M}(0) = \left(r_I D\bar{Q}_I\right)^{\rho\left(1+\frac{1}{\delta_1+e^{\delta_2}}\right)} \tag{20}$$

If $Q_I = \bar{Q}_I \left(= \frac{B}{r_I}\right)$ it immediately follows from (19) that:

$$\hat{M}\left(\bar{Q}_I\right) \equiv \hat{M}_I = 0 \tag{21}$$

When there are no weapon systems, there is nothing to integrate, hence the objective function reaches zero. Since $\hat{M}_0 > 0$, $\hat{M}_I$ is not a potential optimal solution to this optimization problem.

The case of an internal maximum is more complicated. The objective function is not globally concave, so an internal maximum does not necessarily exist, and if it does exist it is not necessarily unique. Even if it exists and is unique, it may not be a global one. There could be a case in which some level of integrative R&D yields a local maximum, but the military capability at that point would still be lower than in the corner case of $Q_I = 0$.

Proving the existence and uniqueness of the optimal solution is based on the following intuition: The objective function can be shown to decrease towards the corner case of $Q_I = \bar{Q}_I$, in which $\hat{M} = 0$. If there exists a point in which the objective function can be shown to increase (i.e. the marginal benefit is higher than the marginal cost), then an internal maximum must exist. When looking for such a point, in which the marginal operational benefit is higher than the marginal operational cost, a natural candidate to look at is the point, denoted Q_I^*, where the difference between the marginal benefit and the marginal cost reaches a maximum.

Proposition 2: Under some mild regularity conditions (provided in the appendix), there **exists** a **unique** Q_I^*.

Proof: See Appendix for the proof and derivation of Q_I^*.

If the objective function increases in this point, then there exists a point where the marginal benefit is higher than the marginal cost, and an internal solution exists. In contrast, if at this point the objective function decreases, then by definition there is no other point where it can increase, and an internal solution does not exist. Proposition 3 formalizes this line of logic, and provides a first order condition for the optimal solution.

Proposition 3:

a. An internal (locally) optimal solution $\hat{Q}_I$ to the program (19) **exists** if and only if:

$$Q_I^* + \frac{1}{\delta_3}\frac{\exp\left(\delta_2 - \delta_3 Q_I^*\right) + 2}{\exp\left(\delta_2 - \delta_3 Q_I^*\right) - 2} < \bar{Q}_I \tag{22}$$

Where Q_I^* is defined in the Appendix.

b. $\hat{Q}_I$ is the **unique** solution to the following equation:

$$\begin{aligned} &\frac{1}{\delta_3}\left[2\exp\left(\delta_3\hat{Q}_I - \delta_2\right) + \exp\left(\delta_2 - \delta_3\hat{Q}_I\right) + 3\right] = \left(\bar{Q}_I - \hat{Q}_I\right)\ln\left[r_I D\left(\bar{Q}_I - \hat{Q}_I\right)\right] \\ &\hat{Q}_I \in \left[Q_I^*, \bar{Q}_I - \tfrac{1}{r_I D}\right] \end{aligned} \tag{23}$$

c. This internal solution is globally optimal if:

$$\hat{M}\left(\hat{Q}_I\right) > \hat{M}_0 \tag{24}$$

(Where $\hat{M}_0 = \hat{M}\left(0\right)$, as defined in Equation (20) above)

Proof: See Appendix.

Equation (23) is the key to the understanding of Proposition 3: The LHS of the equation is inversely proportional to the marginal operational cost[19] of Q_I, whereas the RHS of the equation is inversely proportional to the marginal operational benefit of Q_I. At the optimum, $\hat{Q}_I$, the marginal benefit equals the marginal cost (the LHS equals the RHS). The equation itself cannot be solved analytically, but the existence and uniqueness of the solution allow for a straightforward numerical solution. Condition (22) requires the marginal benefit to be higher than the marginal cost at Q_I^*. Finally, Condition (24) simply states that the internal optimum is a global one if it exceeds the corner solution of $Q_I = 0$.

[19] It is emphasized once more, that the "cost" and "benefit" are measured in terms of (the natural logarithm of) military capability. Inversely proportional measures were used since they significantly simplify the mathematical expressions, while preserving the intuition of marginal analysis.

5. Analysis

This section characterizes the optimal level of integrative technologies. First, it seems reasonable that the benefit from integrative technologies is larger the more systems there are and, hence, the more there is to integrate. Specifically,

Proposition 4: For the case of symmetric weapon systems (i.e. $c_i, r_i, \alpha_i, \beta_i$ and η_i are identical for all i), it holds that:

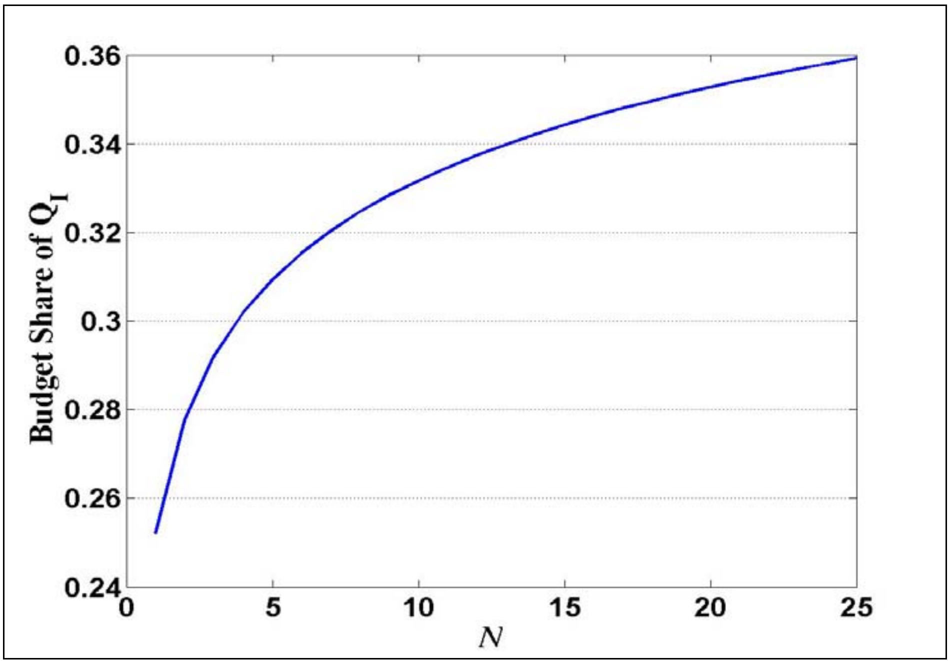
$$\frac{\partial \hat{Q}_I}{\partial N} > 0 \tag{25}$$

Where N is the number of systems.

Proof: See Appendix.

Figure 4 illustrates the dependence of allocation to integrative technologies on the number of systems, for a given set of parameters.

Figure 4: The budget share of integrative technologies as a function of the number of types of systems



Second, integrative technologies are useful when there are sufficient systems to integrate. Moreover, the effect of these technologies on capability is of an S-shape type. Hence, intuition suggests that the benefits from integrative technologies are sizable only if the investment in them is not too small. **Proposition 3** showed that with the S-shape assumption an internal optimal solution for Q_I is not guaranteed. **Proposition 5** shows the dependence of the existence of an internal solution on the budget level; specifically, for sufficiently high budget levels, there is a globally optimal internal solution.

Proposition 5:

There exists a "trigger" value of the budget, denoted B^*, such that:

$$B \geq B^* \Leftrightarrow \hat{Q}_I > 0 \tag{26}$$

Proof: See Appendix.

If some regularity condition holds, the optimal value of $\hat{Q}_I$ experiences a discontinuous jump at B^*, as shown in the following corollary:

Corollary 5.1:

If
$$D < \frac{\delta_3\left(e^{\delta_2} - 2\right)}{r_I\left(e^{\delta_2} + 2\right)} \exp\left(e^{\delta_2} - 2e^{-\delta_2} - 1\right) \tag{27}$$

Then there exists a "trigger" budget, denoted B^*, such that:

$$\begin{cases} \hat{Q}_I = 0 & if \;\; B \leq B^* \\ \hat{Q}_I > Q_I^* > 0 & if \;\; B > B^* \end{cases} \tag{28}$$

Proof: See Appendix.

Condition (27) is met when the initial level of integrative technologies is low (i.e., δ_2 is high), relative to the contribution of the other weapon systems (represented by D). When this is the case, a small expenditure on integrative technologies will have a small contribution to military capability, while expenditure on other weapon systems will have a significant contribution. Hence, when the budget level is small (below the trigger, B^*), the optimal level of integrative technologies is zero; when the budget level is sufficiently high the optimal level of integrative technologies is sizable.

Figure 6 illustrates Corollary 5.1. It shows the military capability as a function of the budget level, B. It can be seen that up to a certain budget level, an internal solution for Q_I doesn't exist. Even when it does exist, at first it is not a global optimum. Only at some higher budget level the internal optimum is a global one. Figure 7 shows the optimal levels of $\hat{x}_1, \hat{q}_1$ and $\hat{Q}_I$ as a function of budget (the values of other $\hat{x}_i, \hat{q}_i$ were also calculated but are not shown, as they behave similarly to $\hat{x}_1$ and $\hat{q}_1$). It can be seen clearly in Figure 7 that the optimal levels exhibit a discontinuous "jump" at the point of transition from a corner optimal solution to an internal one. In particular, the optimal value of $\hat{Q}_I$ jumps from 0 to 4.8. Therefore, in this case it is never optimal to spend less than $4.8 \cdot r_I$ on integrative R&D.

Figure 6: The value of the objective function as a function of budget for the corner solution, $\hat{M}_0$, and the internal solution, $\hat{M}_{INT}$ (when exists)

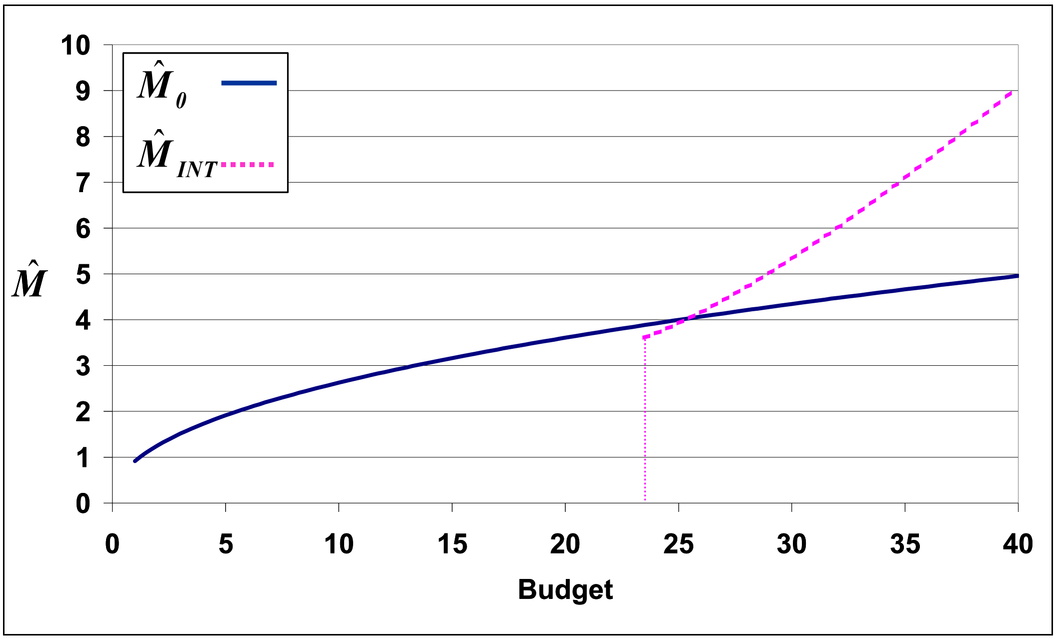
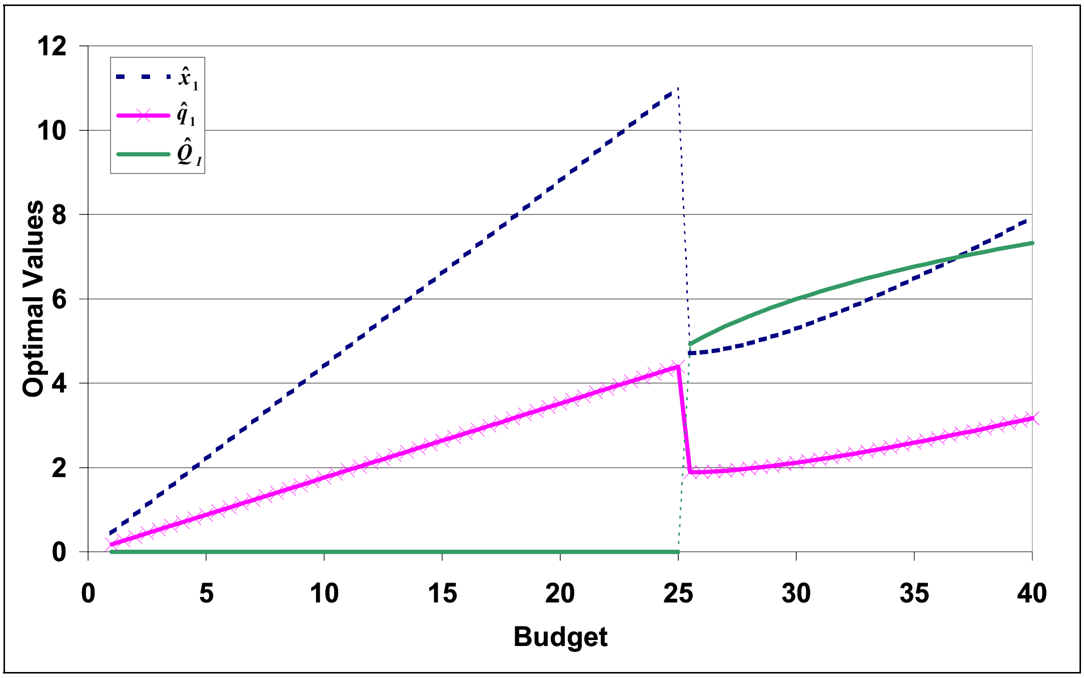


Figure 7: Optimal values of x_1, q_1 and Q_I as a function of budget



Finally, Proposition 6 provides some comparative statics results for the internal optimal solution:

Proposition 6:

The optimal internal solution $\hat{Q}_I$ is

1) **Increasing** in B.
2) **Decreasing** in c_i, r_i and r_I.
3) **Increasing** in β_i (and **decreasing** in α_i) if $\frac{\beta_i}{r_i} > \frac{\alpha_i}{c_i}$.
4) **Increasing** in δ_2 if $\hat{Q}_I > \frac{\delta_2 - \ln\sqrt{2}}{\delta_3}$.

Proof: See Appendix.

The dependence of $\hat{Q}_I$ on B is quite intuitive: once the budget crosses the "trigger" level (so an internal solution is optimal), any additional increase in budget results in an increase of $\hat{Q}_I$. The dependence of $\hat{Q}_I$ on r_I is also intuitively clear - the higher is the cost of integrative technologies, the lower is the optimal level. The fact that $\hat{Q}_I$ is decreasing in r_i (and c_i) is a result of the strong *complementarity* between the weapon systems and the integrative technologies (when there are no weapon systems at all, $\hat{x}_i, \hat{q}_i = 0$, integration is worthless).

When β_i is relatively low (as indicated by the specified condition), an increase of it causes the optimal level of $\hat{q}_i$ to be higher, at the expense of both $\hat{Q}_I$ and $\hat{x}_i$. This behavior changes after β_i is sufficiently large, in which case additional increase in β_i allows to increase the expenditure on $\hat{Q}_I$.

The behavior of $\hat{Q}_I$ as a function of δ_2 depends on their relative values. When $\hat{Q}_I$ is high, and the contribution of integrative R&D is significant, an increase in δ_2 (i.e. a lower initial integration level) would result in increased $\hat{Q}_I$ that would maintain a high final level of integration. However, if $\hat{Q}_I$ is small, then the contribution of integrative R&D is rather small, and a lower initial level would results in decreased expenditures on integrative R&D.

6. Application

This section applies the model to the US military. This choice was motivated by three reasons: first, the US military spends significantly more than any other nation in the world on defense (CDI ,2002), and particularly on defense R&D (SIPRI, 2003). Second, the US military is the world leader in developing and using information age technologies. Last, the availability of US defense budget data, which is published on the Internet, is significantly better than that of any other country.

The case of the US military is analyzed using data for 1996-2005. Various raw data sources are used, and processed, to calibrate the parameter values[20]. Table 1 summarizes all the data sources used, and relates to each source the raw data items it provided.

[20] For brevity, the paper provides highlights of the calibration process. A full account of the calibration method appears in Setter (2004).

Table 1: Summary of data sources for the US defense budget

Source	Data items
National Defense Budget Estimates, (USD-C, 1997a-2003a)	Past appropriations[21] by branch and title, price indices.
RDT&E[22] Programs ("R-1"), (USD-C, 1997b-2003b)	Appropriations by Program Element
Procurement Programs ("P-1"), (USD-C, 1997c-2003c)	Appropriations and quantities by Program
R&D Descriptive Summaries (RDDS, 2003)	Descriptions of Programs by Program Elements
Bureau of Labor Statistics (BLS, 2003)	Consumer Price Index, Employment Cost Index
Science & Engineering Indicators (National Science Board, 2002)	Annual Salaries in scientific and engineering occupations

The data processing included 3 major steps: Labeling of procurement and R&D budgets to the model's categories, estimation of prices and estimation of existing stocks before the first period.

Labeling of budget items: The application follows the budget documents and aggregates the systems to the service level, i.e. Army, Navy and Air Force (N=3). It further distinguishes between "weapon specific" technologies and "integrative"

[21] Appropriations are the sums of money that the US Congress authorizes the DoD to contract upon, for each budget item. We assume that appropriation data provide a good approximation for direct expenditures. Data availability and reliability are the main reasons for using appropriations data.

[22] RDT&E stands for Research, Development, Testing and Evaluation. For consistency with previous sections, we continue using the expression "R&D", taking it to include testing and evaluation activities.

technologies[23]. Such distinction does not appear in the budget documents, and was therefore done on a program-by-program basis. Throughout the entire 10 years period examined (1996-2005), there are approximately 1,000 different R&D Program Elements[24]. Each was labeled as "weapon specific", "integrative", "defense-wide" or "miscellaneous[25]", based on it's title and, in some cases, on the descriptive summary provided in RDDS (2003).

The result of this process is a comprehensive database, which contains all the program elements for the years 1996-2005, each with description, responsible branch or agency, appropriation and technology type. Figures 8 and 9 summarize the aggregate appropriations for procurement and R&D that were used later on in estimating the model's parameters; all figures are in constant 2004 dollars. These appropriations account for more than 75% of the R&D and procurement budget (the remaining 25% are only from the defense-wide budget, so that all the branches' budgets were categorized).

[23] The model also distinguishes among the three services (army, navy, air force) and, for each service, between procurement and R&D (which are given explicitly in the budget document).

[24] The budget provides data at the "Program Element" (PE) level. Each program element may include several related projects.

[25] This category was used for appropriations to intelligence, ballistic missile defense and anti-proliferation agencies.

Figure 8: Annual procurement appropriations by services (in constant 2004 $B)

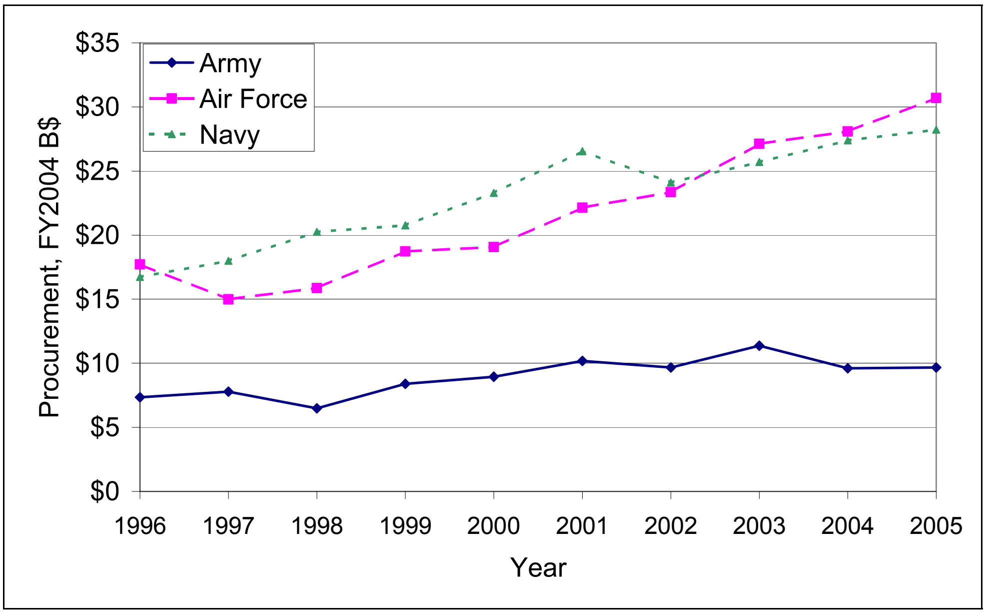
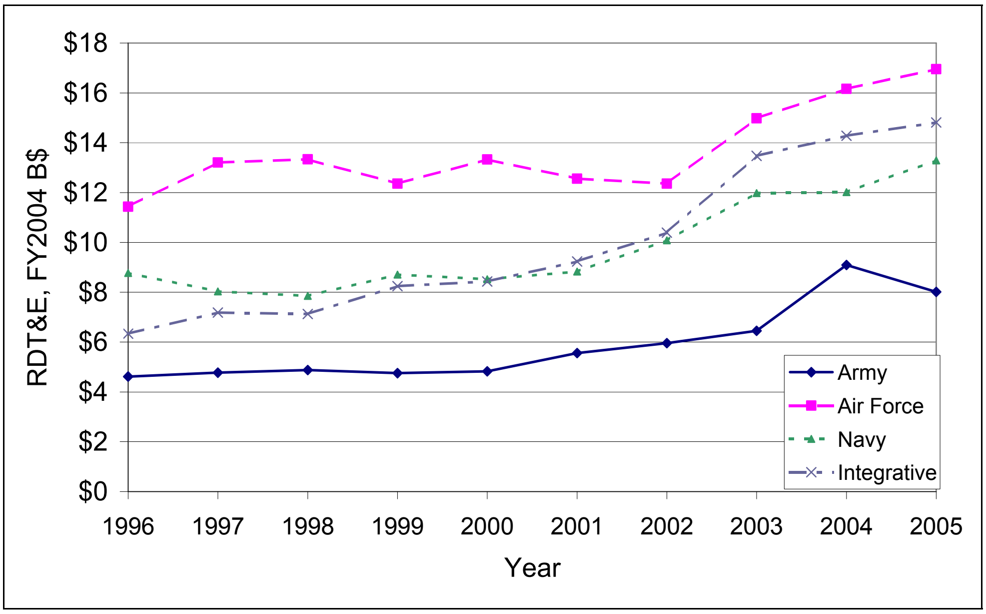


Figure 9: Annual R&D appropriations by services and integrative technologies (in constant 2004 $B)



Estimation of prices: The cost of R&D is driven mostly by scientific and engineering effort, so the median annual salary of scientists and engineers was estimated and used as a price index. The price of an aggregate weapon system (for each of the three services) was estimated by constructing a quality-adjusted price index. The index is constructed using detailed acquisition data, which describes the unit prices and quantity of major equipment for each service during the relevant time period (1996-2005). The index is thus based on a weighted average of the price of different types of equipment, based on their weight in the overall acquisition budget.

Estimation of existing stocks: The budget documents provide detailed appropriations data for every year since 1945, per branch and type of activity (e.g., procurement, R&D). This data, coupled with the price data, was used to assess existing weapons and R&D stocks. Due to the major technology shift occurred after the end of the cold-war era, stocks accumulated prior to 1985 were neglected.

In addition, depreciation rates of 9% for procured equipment and 12% for R&D were used (based on Olvey at al., 1984; Trajtenberg, 2003; Wolf et al., 2000; Wolf and Zycher, 2001) and data characteristics. Depreciation of equipment is due to use, wear and technology obsolescence, while depreciation of technology stocks is mainly due to obsolescence.

The result of the abovementioned processing is a data table, which contains all the model parameters that are observable. Because of the complexity of the model, the calibration of the rest of the parameters is done sequentially, in three stages, and not simultaneously. First, we estimate the annual elasticities for each branch and activity (i.e., α_{army}, β_{army}, etc.). This is done using Equation (29), directly based on Equation (8), which provides the optimal allocation between procurement and R&D for each weapon system:

$$\frac{\alpha_{it}}{\rho} = \frac{c_{it} x_{it}}{B_t - r_I Q_{It}} \qquad (29)$$

Where α_{it}/ρ is the elasticity of procurement of system i, c_{it} is the unit price of system i, x_{it} is the quantity procured of system i, B_t is the budget level, and $r_I Q_{It}$ is the expenditure on integrative technologies (the product of the unit price and the quality level). That is, $B_t - r_I Q_{It}$ is the per-period budget for acquisition and R&D of weapon systems (excluding integrative systems).

Second, the parameters governing the budget allocation between branches are estimated on an annual basis (i.e. ρ, η_{army}, η_{navy}, $\eta_{air\text{-}force}$). Equation (30) is used in this case, based on Equation (6), which specifies the optimal allocation of the budget between the various weapon systems:

$$SH_{it} = \frac{\left(\eta_{it} D_{it}^{\rho}\right)^{\frac{1}{1-\rho}}}{\sum_{i=1}^{N}\left(\eta_{it} D_{it}^{\rho}\right)^{\frac{1}{1-\rho}}} \tag{30}$$

Where SH_{it} is the budget share allocated to system i at time t, η_{it} is the normalizing factor for system i, and D_{it} is the inverse production coefficient of system i (as explained in Section 3).

Finally, a non-linear regression procedure is used to estimate the parameters of integrative technologies (δ_1, δ_2 and δ_3) from Equation (31), which is based on Equation (23), the first order condition for the optimal value of Q_I:

$$\frac{1}{\delta_3}\left[2\exp\left(\delta_3\hat{Q}_{It} - \delta_2\right) + \exp\left(\delta_2 - \delta_3\hat{Q}_{It}\right) + 2\delta_1 + 1\right] = \left(\bar{Q}_{It} - \hat{Q}_{It}\right)\ln\left[r_{It} D\left(\bar{Q}_{It} - \hat{Q}_{It}\right)\right] \tag{31}$$

Where $\bar{Q}_{It}$ is the maximum potential stock of integrative technologies (i.e. if the entire budget was used only to develop integrative technologies); $\hat{Q}_{It}$ is the stock of integrative technologies; δ_1, δ_2 and δ_3 are the parameters of the S-shape contribution function of integrative technologies[26] (as defined in Section 4).

Table 2 shows the estimated values of the parameters of integrative technologies, along with asymptotic *t*-values and confidence intervals. These results are robust for different starting values of the regression.

Table 2: Estimated values of the parameters of the S-shaped contribution function

Parameter	Estimated Value	t value	Confidence Interval (95%)
δ_1	8.53*	31.7	(7.92 , 9.13)
δ_2	2.83*	33.9	(2.70 , 2.97)
δ_3	1.13*	12.2	(1.05 , 1.21)

* Significance level <0.01

Figure 10 plots the integrative technologies improvement profile, using the calibrated parameter values, as a function of R&D stocks. It shows the S-shape curve with the calibrated values of the δ_i's. The annual stocks of integrative technologies (as calculated by the model) are plotted as small squares on the curve to illustrate the advancement made during the last decade. Recall that $\frac{1}{\delta_1}$ is the asymptotic value of the improvement function, so the integrative capability represented by the 2005 stock of integrative technologies achieves approximately 60% of potential integrative capability.

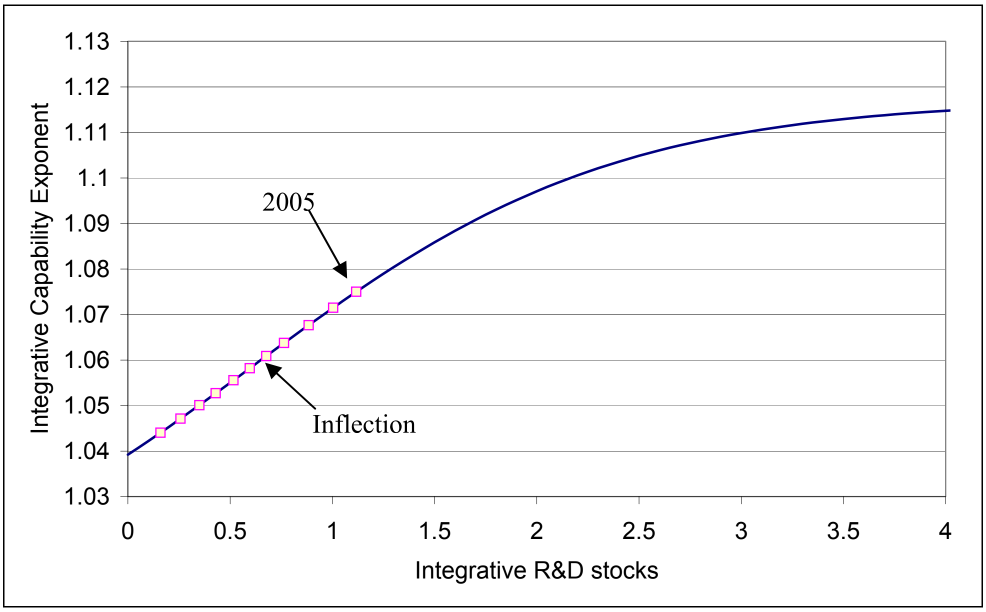
Furthermore, it is possible to estimate the location of the inflection point (denoted $\tilde{Q}_I$), using the following expression, based on expression (16):

[26] For expositional purposes, Chapter 5 assumed δ_1=1. The derivation of the first order condition for any δ_1 is straightforward.

$$\tilde{Q}_I = \frac{\delta_2 - \ln(\delta_1)}{\delta_3} = 0.67 \qquad (32)$$

Thus, it seems that integrative technologies featured a convex contribution up until 2001 (the year in which Q_I crossed the value of 0.67). Starting in 2002 the marginal effect of Q_I on integrative capability, $g(Q_I)$, is decreasing.

Figure 10: The contribution function of integrative technologies, based on the estimated parameter values (the estimated evolution of integrative technologies stocks is plotted as squares)



7. Conclusion

Expenditure on defense R&D activities constitutes a major share of defense R&D budgets. Consequently, the allocation of budget to defense R&D, particularly in light of the recent emergence of integrative technologies, poses an important and complicated decision problem. The scarce existing literature does not address the unique role of integrative technologies, namely a "military-wide" effect, a "network" effect and an *S*-shaped returns profile.

This study aims to establish an analytic framework addressing this decision problem, to analyze the effect of integrative technologies on budget allocation decisions, and to apply the theory to real-world data. To meet these goals, the study employs a variety of analytical, computational and empirical methods.

The study develops an analytical framework that captures the defining characteristics of integrative technologies, and provides a simple budget allocation procedure, which yields, under some mild regularity conditions, a unique optimal solution. It proves that the optimal expenditure on integrative technologies depends on the number of weapon systems, and on the capability of each system. Furthermore, integrative technologies are shown to complement other system-specific technologies.

The study applies the theoretical framework to the US military. The calibration of the parameters shows that the US military has recently moved from the "increasing returns" part of the military contribution of integrative technologies into the "decreasing returns" part. Still, sizable expenditures on integrative technologies are expected for the next 5-10 years or so.

The study also demonstrates the existence of a budget threshold for the expenditure on integrative technologies, because of their S-shaped returns profile: if the budget level is below the threshold, it is not optimal to spend *any* money on integrative technologies; Above the threshold, on the other hand, a sizable expenditure is optimal. Accordingly, the expenditure on integrative technologies is *not* a continuous function of the budget. This result implies that integrative technologies are expected in countries whose defense budgets are relatively large (USA, some Western European countries, Israel, etc.). Other countries, either poor or with a low technological level (such as most African and Arab countries), are not expected to invest in such technologies.

The central limitation of the model is its sensitivity to the level of aggregation of the analysis, because of the characteristics of the chosen functional form. Furthermore, the model's variables are not directly observed, and thus depend on the unit of measurement. The optimal military capability changes when the analysis becomes more (or less) aggregated; hence, care must be exercised when using it for specific applications. The choices of aggregation level (e.g., service level, as in this study) and units of measurement must rest on a thorough understanding of the structure and mode of operation of the analyzed military. Similarly, comparison of results between the military capabilities of different military forces should be done cautiously.

Furthermore, the application of the model was limited to unclassified publicly available US military data. The data used on the US military may thus be subject to some inaccuracies and deficiencies. Application of the model to other countries, which do not publish detailed defense data to the public, is bound to be significantly more complicated.

The study offers numerous possible future research paths. First, the model may be applied to other countries. A second possible extension is to allow for a model of several hierarchical levels, where integration may occur within a certain level or between varying levels. Third, a future model may incorporate strategic interactions of three types: interactions between rival nations (Intriligator and Brito, 2000), interactions between allied nations, and internal interactions within a nation's defense establishment. Finally, the research may be extended to civilian applications of integrative technologies. Such technologies are prevalent in modern business organizations (e.g. email, videoconferencing, Knowledge Management systems, Enterprise Resource Planning systems), and exploring their effect on organizational strategy and market structure may be an interesting line of research.

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