Managing Asymmetric Conflict*

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Abstract

The paper proves that the possibility of differentiation from an incumbent power's technology by a contestant group makes it harder for the incumbent power to want to implement peace by means of effort. In some cases, it may make it altogether unfeasible. The incumbent power is then more likely to adopt a defensive strategy to cope with a defensive type of conflict. This strategy actually involves less effort that the one that would be made if a defensive strategy was chosen to be induced in the absence of possible differentiation by the incumbent.

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Contents

1 Introduction

In the traditional arms race and conflict literature (Siqueira, 2002 offers a recent example within the conflict literature), the "technology of conflict" is seen as a unidimensional variable. A higher amount of effort, translated into more or more advanced weapons, is the only technological decision conflicting parties take. Reality though does not seem so simple. Technological decisions seem to be multidimensional. Not only effort/quality is chosen but also the type of weaponry and training which conflict groups acquire. This results in a potential asymmetry of technologies actually being used.

The objective of this paper is to present a simple model that captures the main features and interactions involved in a situation where the use of asymmetric conflict technologies can arise.

Asymmetric conflict is most likely to arise when conflicting powers are themselves asymmetrical in nature. A established power, with a lead in available resources but committed to a certain type of technology and a contestant power, with limited resources but more flexibility in choice of type of conflict technology. Most incumbent powers one could think of tend to have military capabilities whose characteristics are very similar. Actually, when similar powers engage in conflicts or arms escalation they do so with very similar technologies. The Cold War, for instance, was mainly a vertical arms race, i.e., same type technology with effort/quality being the main strategic variable. Having invested so much in a certain conflict technology and the industry to produce it and export, incumbent countries might find it difficult to change it in response to security challenges. A second factor that may explain persistence of conflict technologies may be that it is seen as the most suitable technology to fight the "average threat" in a Post Cold War era where potential conflicts are not so easily foreseen. Contestant powers do not have this problem though, their constrained resources and dependency on traditional weapon imports may actually encourage them to find new ways of fighting, that could become an advantage in certain types of conflicts. Actually, it may be the very availability of resources that established powers enjoy, the encouragement for contestants to design new potentially dangerous conflict technologies.

The above situation resembles the ways in which firm competition has been evolving in the computer industry as illustrated by Bresnahan and Greenstein (1999) and Sutton

(2001). During the 1990's IBM was the clear main player in the computing industry, having for long had a way of doing business and a well established customer base. However, in a few years the industry would change dramatically, "instead of competing directly with the established leader, entrants opened up new market segments", later, entrants became challengers to the IBM's power and those who in the end prevailed where those who won the standardization war that was fought, Microsoft and Intel. This outcome was not ex-ante certain, the uncertainty component is also present in military conflict.

Some of the characteristics of asymmetric warfare can be captured using a simple model, which introduces a variation to the traditional conflict model in order to make it bidimensional and therefore allow for asymmetric conflict technologies. Probabilities of winning conflicts are generally increasing functions of owns effort and decreasing functions of the opponent's effort. We keep this structure but we introduce an additional parameter in the probability functions representing horizontal technological differentiation from an incumbent power.

The probability of winning a conflict –the Contest Success Functions– will depend on who attacks and who needs to defend itself. If only one party attacks (the other acting defensive), the attacking party will have an advantage whenever the technology used for attach is different from the technology that the defending party has. The idea here is that if you decide to attack with a missile, you will have an advantage (for same given efforts) if the defensive party does not have an advance missile technology as they are not likely to have antimissile systems, same example with biowarfare. In addition, asymmetric technology gives the contestant party an advantage if it is on the offensive. However if it is on the defensive, technology differentiation becomes a liability.

Incumbent and contestant's interaction is represented by the following multiple stage game. Throughout the game, the constant's effort is exogenous and so it is the technology type of the incumbent. The first assumption is based on them just using all or most of their available funds for the conflict, while incumbents are assumed to have more flexibility in the use of their funds, this seems to fit the problem of established powers and contestant groups. The structure of the game is as follows, in stage1, the incumbent chooses effort. Contestant's effort is considered exogenous an based of them just using all their funds, while incumbents allocate country's budget across other expenditures and therefore have more flexibility in choosing effort.

In stage 2, the contestant chooses the type of technology to be acquired. Technological differentiation from the contestant is represented by a variable which increases as technological differences between the contestant and incumbent are wider and takes a value of zero if they are exactly the same type (even though they might be of different qualities, which is captured by the effort variable). Technology types could range from atomic devices, biological warfare, conventional weapons, internet warfare, etc. Finally, in stage 3, incumbent and contestant simultaneously decide whether to adopt an attack or defense strategy. The probabilities of winning the war will be determined by who attacks and who defends. If only one of the two parties adopts the attack strategy, such party will have an advantage with respect to the defensive party if its technology is different to that of the defending party. If both parties adopt an attack strategy, technological differences will cancel each other out and success probabilities will be solely based on effort levels. If nobody attacks, they have a exogenously given share of resources, which could also represent an agreed share of resources by means of peaceful negotiation.

We find the Subgame Perfect Equilibrium of the above described game using backwards induction.

A few assumptions are adopted for purposes of solving the model. First, we assume that some of the resources are destroyed through conflict, destruction being higher in a conflict in which both parties adopt an attack strategy (in what follows, we will refer to this as mutual attack). Second, we assume that the incumbent's share of resources if no conflict erupts is bigger than the expected resources to be won through conflict. The opposite is assumed for the contestant.

Already with this simple model a few results can be obtained. Under the model's assumptions and for a small trenching advantage, a mutual attack strategy becomes a possibility only once we introduce technological differentiation. However, the contestant will block mutual attack equilibrium by limiting technological differentiation. Given this, two possibilities remain candidates for the equilibrium in the last stage of the game: no conflict and a conflict where the incumbent adopts a defensive strategy and contestant adopt and attack strategy. Through choice of effort, the incumbent may be able to implement any of these two equilibria as the unique equilibrium in the final stage of the game.

Interestingly, it may actually be "cheaper" in terms of effort for the incumbent to induce a defensive conflict, specially if the no conflict shares of resources and imbalanced towards the incumbent side. If the incumbent targets a defensive conflict, when the contestant has the option of differentiation from the incumbent, the outcome is a lower level of effort by the incumbent. However, if the incumbent wanted to implement no conflict, when the contestant has the option of differentiation, the level of effort will actually be higher. From here we are able to conclude it is less likely for a incumbent to want to implement no conflict when differentiation becomes a possibility.

Our research is also linked to the terrorism literature. It has been suggested in this literature (see e.g., Enders and Sandler (2002) and Sandler and Arce (2003)) that when a terrorist group has a choice of targets (those targets being different countries or different objectives within the same country) that effort being put into defending one target may actually encourage terrorists to shift to the alternative option. In our paper, the discrete choice of targets becomes a continuous choice of conflict technologies into which the contestant in locked.

The remainder of the paper is organized as follows. Section two introduces the main features of the model. Section three finds the Subgame Perfect Nash equilibrium of the game and compares it with the benchmark case in which technological differentiation is not present. Section 4 concludes the paper.

2 The Model

2.1 Structure of the game

In our model incumbent and contestant "play" a sequential game with the following stages:

Stage 1: Incumbent chooses effort. Contestant's effort is considered exogenous, based on them just using all their funds, while incumbents allocate country's budget across other expenditures and therefore have more flexibility in choosing effort.

Stage 2: The contestant chooses the type of technology to be acquired; the technology used by incumbent is exogenous through out the model. Technological differentiation from the incumbent is represented by a variable which increases as technological differences between the contestant and incumbent are wider and takes a value of zero if they are exactly the same type (even though they might be of different qualities, which is captured by the effort variable). Technology types could range from atomic devices, biological warfare, conventional weapons, internet warfare, etc.

Stage 3: Incumbent and contestant decide simultaneously whether to adopt an attack or defense strategy. The probabilities of winning the war will be determined by who attacks and who defends. If only one of the two parties adopts the attack strategy, such party will have an advantage with respect to the defensive party if its technology is different to that of the defending party. If both parties adopt an attack strategy, technological differences will cancel each other out and success probabilities will be solely based on effort levels. If nobody attacks, they have a exogenously given share of resources, which could also represent an agreed share of resources by means of peaceful negotiation.

We find the Subgame Perfect Equilibrium of the above described game using backwards induction.

2.2 Contest Success Functions

We now define the probabilities of winning a conflict. Note that the first argument in parenthesis always refers to the strategy of the incumbent and the second refers to the contestant's strategy:

$P_a(A, D)$ = probability that incumbent has of winning if they attack and contestant defends.

$P_a(D, A)$ = probability that the incumbent has of winning if the contestant attacks and they defend.

$P_b(A, D)$ = probability that contestant has of winning if it defends and the incumbent attacks.

$P_b(D, A)$ = probability that contestant has of wining if it attacks the incumbent defends.

Similarly define $P_b(D, D)$, $P_a(D, D)$, $P_b(A, A)$ and $P_a(A, A)$. Then the first of our crucial set of assumptions is formalized by the following contest success functions (CSFs):

$$\begin{aligned}
P_a(A,D) &= \frac{t+e_a}{\Phi+t+e_b+e_a} \leq 1. \\
P_a(D,A) &= \frac{\Phi+e_a}{\Phi+t+e_b+e_a} \leq 1. \\
P_a(A,A) &= \frac{e_a}{e_b+e_a} \leq 1. \\
P_a(D,D) &= \textit{status quo incumbent share} = s_a = 1-s_b
\end{aligned}$$

where Φ represents the additional obstacles faced by the contestant given that incumbent is already entrenched (entrenching advantage), t is the technology location of the contestant and e_a and e_b represent the incumbent and contestant's effort respectively. It follows that

$$P_b(D,A) = 1 - P_a(D,A) = \frac{t+e_b}{\Phi+t+e_b+e_a} \leq 1$$

and

$$P_b(A,D) = 1 - P_a(A,D) = \frac{\Phi+e_b}{\Phi+t+e_b+e_a} \leq 1$$

Note that if $\Phi = 0$:

$$\begin{aligned}
P_b(D,A) &\geq P_b(A,A) \geq P_b(A,D). \\
P_a(D,A) &\leq P_a(A,A) \leq P_a(A,D).
\end{aligned}$$

where equalities apply if both sides adopt the same technologies.

2.3 Payoffs

Given stage 3 strategies (S_a, S_b), S_i = (Attack, Defend) by incumbent and contestant respectively, the expected utilities of the incumbent and contestant are:

$$\begin{aligned}
EU_a(S_a,S_b) &= P_a(S_a,S_b)\phi_a(S_a,S_b)V - C_a(e_a). \\
EU_b(S_a,S_b) &= P_b(S_a,S_b)\phi_a(S_a,S_b)V - C_b(e_b).
\end{aligned}$$

where V is a fixed amount of rent or resources that both incumbent and contestant are aiming to control, $e_i, i = a, b$ are resources devoted to fighting with cost $C(e_i)$. As already noted, the effort of the contestant, e_b, is exogenous throughout. Finally, $(1-\phi_a(S_a,S_b))V$ represents the cost of being involved in a conflict for the incumbent and similarly $(1-\phi_b(S_a,S_b))V$ represents the cost of being involved in a conflict for the contestant. If both countries defend, there is no war and these costs are not incurred. A crucial feature in our model is the introduction of a conflict type sensitive cost of conflict. For most of the results presented in this paper it will be enough to assume that the costs of a mutual attack conflict are higher than those of a unilateral conflict. Also, we assume that the incumbent's share of resources if no conflict erupts is bigger than the expected resources to be won through conflict. The opposite is assumed for the contestant.

2.4 The benchmark model without differentiation

In this section, we solve the game for the case when technological differentiation is not possible, we would then have a two stage game where incumbent decides effort in the first stage and incumbent and contestant simultaneously decide whether to attack or defend. The associated winning probabilities are equivalent to those already describe but with $t = 0$.

2.4.1 Last stage equilibria

NASH EQUILIBRIUM 1 :$(ATTACK, ATTACK)$

$$\begin{aligned} \phi_b(A,A)\,P_b(A,A) &\geq \phi_b(A,D)\,P_b(A,D) \\ \phi_a(A,A)\,P_a(A,A) &\geq \phi_a(D,A)\,P_a(D,A) \end{aligned} \tag{1}$$

This is not a Nash Equilibrium based on the assumption $\phi_a(A,A) < \phi_a(D,A)$.

NASH EQUILIBRIUM 2: $(DEFEND, DEFEND)$

$$\begin{aligned} s_b &\geq \phi_b(D,A)\,P_b(D,A) \\ s_a &\geq \phi_a(A,D)\,P_a(A,D) \end{aligned} \tag{2}$$

This occurs if the combination e_a satisfies

$$\begin{aligned} P_b(D,A)\phi_b(D,A) &= \frac{e_b\phi_b(D,A)}{\Phi + e_b + e_a} \leq s_b \\ P_a(A,D)\phi_a(A,D) &= \frac{(\Phi + e_a)\phi_a(A,D)}{\Phi + e_b + e_a} \leq s_a \end{aligned}$$

Assuming: $\phi_a(A,D) < s_a$, we just need:

$$\frac{e_b\phi_b(D,A)}{\Phi + e_b + e_a} \leq s_b$$

$$\frac{e_b\phi_b(D,A) - s_b\left(\Phi + e_b\right)}{s_b} = \frac{e_b\left(\phi_b(D,A) - s_b\right) - s_b\Phi}{s_b} \leq e_a.$$

$$e_a^{(D,D,t=0)} = \frac{e_b\left(\phi_b(D,A) - s_b\right) - s_b\Phi}{s_b}$$

In this case, $(DEFEND, DEFEND)$ will be the unique Pure Strategy Nash Equilibrium.**NASH EQUILIBRIUM 3:** $(DEFEND, ATTACK)$

$$\begin{aligned} s_b &\leq \phi_b\left(D,A\right)P_b\left(D,A\right) \\ \phi_a\left(D,A\right)P_a\left(D,A\right) &\geq \phi_a\left(A,A\right)P_a(A,A) \end{aligned} \tag{3}$$

Note that $\phi_a\left(D,A\right)P_a\left(D,A\right) > \phi_a\left(A,A\right)P_a(A,A)$ due to our assumptions. Therefore, this will be the unique Pure Strategy Nash Equilibrium as long as $s_b \leq \phi_b\left(D,A\right)P_b\left(D,A\right).$

NASH EQUILIBRIUM 4: $(ATTACK, DEFEND)$

$$\begin{aligned} s_a &\leq \phi_a\left(A,D\right)P_a\left(A,D\right) \\ \phi_b\left(A,D\right)P_b\left(A,D\right) &\geq \phi_b\left(A,A\right)P_b\left(A,A\right) \end{aligned} \tag{4}$$

We reject it by assuming $s_a > \phi_a\left(A,D\right)$.

2.4.2 Optimal level of effort without differentiation

We first define $e_a^{(D,D,t=0)}$ and $e_a^{(D,A,t=0)}$ as the levels of effort the contestant would make depending on whether (D,A) or (D,D) was the target. First note that for (D,D) to be induced the incumbent would need to ensure at least $s_b = \phi_b\left(D,A\right)P_b\left(D,A\right).$ If $s_b < \phi_b\left(D,A\right),$ some effort will be required to ensure that

$$e_a^{(D,D,t=0)} = \frac{e_b\left(\phi_b(D,A) - s_b\right)}{s_b}.$$

If on the other hand, the incumbent wanted to induce (D,A) would put effort that maximizes $EU_a\left(D,A\right)$.

$$\frac{dEU_a(D,A)}{de_a} = V\frac{e_b\phi_a(D,A)}{(\Phi+e_b+e_a)^2} - \frac{dC(e_a)}{de_a} = 0.$$

For $\frac{dC(e_a)}{de_a} = c$ and $\Phi = 0$

$$e_a^{(D,A,t=0)} = \sqrt{V\frac{\phi_a(D,A)\,e_b}{c}} - e_b.$$

The decision of the government of whether to induce (D,A) or (D,D) will depend crucially on the amount of effort required to prevent conflict relative to the other effort. We will discuss this in depth in the following section.

3 Subgame Perfect Equilibrium of the Game with Differentiation

We now obtain the Subgame Perfect Equilibrium of the game in which differentiation is possible using backwards induction.

3.1 Stage 3: Simultaneous choice of defend or attack

The payoff matrix at third stage of the game is given by:

$a\backslash b$	$ATTACK$	$DEFEND$
$ATTACK$	$P_a(A,A)\phi_a(A,A)V,\ P_b(A,A)\phi_b(A,A)V$	$P_a(A,D)\phi_a(A,D)V,\ P_b(A,D)\phi_b(A,D)V$
$DEFEND$	$P_a(D,A)\phi_a(D,A)V,\ P_b(D,A)\phi_b(D,A)V$	$s_aV,\ s_bV$

Note that, the costs of effort do not appear in the payoff matrix because they are considered sunk at this stage.

We now consider the four possible candidates for Pure Strategy Nash equilibria in the last stage of the game.

NASH EQUILIBRIUM 1 :$(ATTACK, ATTACK)$

$$\begin{aligned}\phi_b(A,A)\,P_b(A,A) &\geq \phi_b(A,D)\,P_b(A,D)\\ \phi_a(A,A)\,P_a(A,A) &\geq \phi_a(D,A)\,P_a(D,A)\end{aligned} \tag{5}$$

Note that, there must be a minimum of technological differentiation for this equilibrium to happen, given our conjecture that $\phi_a(A,A) < \phi_a(D,A)$. But, now even if $\phi_a(A,A) = \phi_a(D,A)$ we still need technological differentiation for that equilibrium to come up due to Φ.

NASH EQUILIBRIUM 2: $(DEFEND, DEFEND)$

$$\begin{aligned} s_b &\geq \phi_b(D,A) P_b(D,A) \\ s_a &\geq \phi_a(A,D) P_a(A,D) \end{aligned} \tag{6}$$

In this case, $(DEFEND, DEFEND)$ will be a Pure Strategy Nash Equilibrium.

NASH EQUILIBRIUM 3: $(DEFEND, ATTACK)$

$$\begin{aligned} s_b &\leq \phi_b(D,A) P_b(D,A) \\ \phi_a(D,A) P_a(D,A) &\geq \phi_a(A,A) P_a(A,A) \end{aligned} \tag{7}$$

In this case, $(DEFEND, ATTACK)$ will be Pure Strategy Nash Equilibrium. Note that this equilibrium could happen together with $(ATTACK, ATTACK)$ if $\phi_a(D,A) P_a(D,A) = \phi_a(A,A) P_a(A,A)$. Note also that, if $\phi_a(A,A) = \phi_a(D,A)$, $P_a(D,A) > P_a(A,A)$, due to Φ. Therefore, this equilibrium will arise without technological differentiation as long as $s_b \leq \phi_b(D,A) P_b(D,A)$.

NASH EQUILIBRIUM 4: $(ATTACK, DEFEND)$

$$\begin{aligned} s_a &\leq \phi_a(A,D) P_a(A,D) \\ \phi_b(A,D) P_b(A,D) &\geq \phi_b(A,A) P_b(A,A) \end{aligned} \tag{8}$$

In this case, $(ATTACK, DEFEND)$ will be Pure Strategy Nash Equilibrium. Note that this equilibrium could happen together with $(ATTACK, ATTACK)$ as long as $\phi_b(A,D) P_b(A,D) = \phi_b(A,A) P_b(A,A)$.

Also, again note that, if $\phi_b(A,A) = \phi_b(D,A)$, $P_b(D,A) > P_b(A,A)$, due to Φ. Therefore, this equilibrium will arise without technological differentiation as long as $s_a \leq \phi_a(A,D) P_a(A,D)$. However, throughout most of the analysis in the following sections we will restrict ourselves to the case in which $\phi_a(A,D) \leq s_a$, therefore, ignoring (A,D) as a possibility.

3.2 Stage 2: Choice of Differentiation

Consider the possibility of an equilibrium type 2, (D, D). From (6) and the CSFs this occurs if the combination (e_a, t) satisfies

$$\begin{aligned} P_b(D,A)\phi_b(D,A) &= \frac{(t+e_b)\phi_b(D,A)}{\Phi+t+e_b+e_a} \leq s_b \\ P_a(A,D)\phi_a(A,D) &= \frac{(\Phi+e_a)\phi_a(A,D)}{\Phi+t+e_b+e_a} \leq s_a \end{aligned}$$

It follows that given e_a, decided in stage 1, equilibrium (D, D) occurs iff

$$t \leq \min\left[\frac{(\Phi+e_a)s_b}{\phi_b(D,A)-s_b} - e_b, \frac{(\Phi+e_b)s_a}{\phi_a(A,D)-s_a} - e_a\right] = [t_1(e_a), t_2(e_a)]\,;\ t_1' > 0\ \ t_2' < 0 \quad (9)$$

say. For high e_a, the $t < t_2$ is then the binding constraint: high product differentiation invites an attacking strategy by incumbent. For low e_a, $t < t_1$ is the binding constraint: the contestant then chooses low product differentiation as part of a defensive strategy.

We could also consider the possibility of an (A, D) equilibrium. Given e_a,by choosing $t < t_2(e_a)$, the contestant can block this possibility. They will wish to do this if $s_b > P_b(A, D)\phi_b(A, D)$; i.e., if

$$t > \frac{(\Phi+e_b)(\phi_b(A,D)-s_b) - s_b(\Phi+e_a)}{s_b} \quad (10)$$

But if $\phi_b(A, D)$ is low, it is plausible to assume that the right hand side of (10) is negative and the contestant will block equilibrium (A, D) by not exceeding the level of technology differentiation that invites an offensive strategy by the incumbent. Note however that if we assume that $\phi_a(A, D) \leq s_a$, (A, D) would never happen and therefore the contestant would never have to consider blocking it.

Now consider equilibria (D, A) and (A, A). The following analysis shows that the contestant will block equilibrium (A, A) by choosing the maximum possible technological differentiation, $t_3(e_a)$, which just falls short of provoking an offensive strategy (A, A); i.e., the contestant will

$$\begin{aligned} &\underset{\{t\}}{Max} \qquad \phi_b(D,A)\,P_b(D,A)\,V = \phi_b(D,A)\,\frac{t+e_b}{\Phi+t+e_b+e_a}V \\ &subject\ to \qquad \phi_a(D,A)\,P_a(D,A) \geq \phi_a(A,A)\,P_a(A,A) \end{aligned}$$

Now, as the contestant's objective function is increasing in t,

$$\frac{\partial P_b(D,A)}{\partial t} = \frac{t(\Phi+t+e_b+e_a)-(t+e_b)}{(\Phi+t+e_b+e_a)^2} = \frac{(\Phi+e_a)}{(\Phi+t+e_b+e_a)^2} > 0,$$

the contestant will simply set the maximum possible technological differentiation, $t = t_3(e_a)$, which is the one that makes the incumbent's constraint bind,

$$\phi_a(D,A)\, P_a(D,A) = \phi_a(A,A)\, P_a(A,A)$$

Substituting above:

$$\phi_a(D,A)\,\frac{\Phi + e_a}{\Phi + t + e_b + e_a} = \phi_a(A,A)\,\frac{e_a}{e_a + e_b} \Longleftrightarrow$$

$$t_3(e_a) = \frac{\phi_a(D,A)\,(e_a+e_b)\,(\Phi+e_a) - \phi_a(A,A)\, e_a\,(\Phi + e_b + e_a)}{\phi_a(A,A)\, e_a}. \tag{11}$$

Note that for (D, A) with the above differentiation to be an equilibrium of the second stage of the game it has to be the case that $\phi_b(A,A)\, P_b(A,A) < \phi_b(D,A)\, P_b(D,A)$ for $t_3(e_a)$, otherwise, it would pay for the contestant to just force the (A, A) equilibrium, still $t_3(e_a)$ would have to be done to force (A, A) equilibrium to happen. It can be seen that if $\Phi = 0$, this would not be an issue as long as $\phi_b(A,A) \leq \phi_b(D,A)$, which was our initial conjecture.

Let's assume $\Phi > 0$.A sufficient condition for (A, A) not to be a possibility is

$$P_b(A,A) < P_b(D,A)|_{t=t_3(e_a)} \Leftrightarrow \frac{e_b}{e_a+e_b} < \frac{t_3(e_a)+e_b}{\Phi + t_3(e_a) + e_b + e_a} \Leftrightarrow t_3(e_a) > \frac{\Phi e_b}{e_a} = t_4(e_a)$$

say. Let's check that the above holds for the $t_3(e_a)$ we obtained:

$$\begin{aligned}
t_3(e_a) = \frac{\phi_a(D,A)\,(e_a+e_b)\,(\Phi+e_a) - \phi_a(A,A)\, e_a\,(\Phi+e_b+e_a)}{\phi_a(A,A)\, e_a} &> \frac{\Phi e_b}{e_a} = t_4(e_a) \Leftrightarrow \\
\phi_a(D,A)\,(e_a+e_b)\,(\Phi+e_a) - \phi_a(A,A)\, e_a\,(\Phi+e_b+e_a) &> \Phi\phi_a(A,A)\, e_b \Leftrightarrow \\
\Phi\phi_a(D,A)\, e_b + e_a\,(\Phi+e_b+e_a)\,(\phi_a(D,A) - \phi_a(A,A)) &> \Phi\phi_a(A,A)\, e_b \Leftrightarrow \\
\Phi e_b\,(\phi_a(D,A) - \phi_a(A,A)) + e_a\,(\Phi+e_b+e_a)\,(\phi_a(D,A) - \phi_a(A,A)) &> 0.
\end{aligned}$$

This as long as $\phi_a(D,A) > \phi_a(A,A)$, which is our conjecture. Therefore, $t_3(e_a)$ will never be low enough as to make the contestant prefer an all out war (in which case technological differentiation would be done with the objective of triggering an mutual attack equilibrium). Note that $t_3(e_a)$ is a convex function of e_a and that the sign of $\frac{dt_3(e_a)}{de_a}$ is ambiguous:

$$\frac{dt_3(e_a)}{de_a} = -\frac{\Phi\phi_a(D,A)\, e_b}{t\phi_a(A,A)\,(e_a)^2} + \frac{(\phi_a(D,A) - \phi_a(A,A))}{t\phi_a(A,A)} > 0 \Longleftrightarrow$$

$$-\frac{\Phi\phi_a(D,A)e_b}{(e_a)^2}+(\phi_a(D,A)-\phi_a(A,A))>0.$$

Therefore, for e_a low enough, increases in effort will decrease technological differentiation. Also, if $\phi_a(D,A)=\phi_a(A,A)$, the effect will be clearly negative, while if $\Phi=0$ the effect is positive as long as our assumption $\phi_a(D,A)>\phi_a(A,A)$ holds. Also note that:

$$\frac{dt_3(e_a)}{d\Phi}>0.$$

Our findings can be summarized in the following proposition:

Proposition 1. *There are two candidates for unique Pure Strategy Nash Equilibrium in the conflict game (D,D) and (D,A). Given e_a, the contestant can induce (D,D) by choosing $t<\min[t_1(e_a),t_2(e_a)]$ and will always choose to block (A,D). If $t_3(e_a)>t_1(e_a)$, the contestant will induce (D,A) and will block (A,A).*

3.3 Stage 1: Incumbent chooses Effort

3.3.1 Incumbent's effort that implements (D,D) as the unique Pure Strategy Nash Equilibrium in the last stage of the game

If the incumbent wants to stop conflict, it will need to ensure:

$$\begin{aligned} s_b &\geq \phi_b(D,A)P_b(D,A), \\ s_a &\geq \phi_a(A,D)P_a(A,D), \end{aligned}$$

through its choice of effort.

We define $e_a^{(D,D)}$ as the lowest level of effort that implements equilibrium (D,D). That is, the lowest level of effort that makes $t_3=t_1$. This is equivalent to finding the effort that makes $s_b\geq\phi_b(D,A)\,P_b(D,A)|_{t=t_3}$.

First note that if $\phi_b(D,A)<s_b$, then (D,D) would happen for sure and no effort would be necessary on the side of the incumbent to ensure it. We assume the opposite and continue.

Now, $t_1=t_3$ (see equations (9) and (11)) iff

$$\frac{(\Phi+e_a)s_b}{\phi_b(D,A)-s_b}-e_b=\frac{\phi_a(D,A)(e_a+e_b)(\Phi+e_a)-\phi_a(A,A)e_a(\Phi+e_b+e_a)}{\phi_a(A,A)e_a}.$$

In general, this results in a nonlinear equation, however, for $\Phi = 0$,

$$e_a^{(D,D)} = \frac{e_b \phi_a (D,A) (\phi_b(D,A) - s_b)}{\phi_b(D,A)\phi_a (A,A) - (\phi_b(D,A) - s_b) \phi_a (D,A)}.$$

Note that:

$$\frac{de_a^{(D,D)}}{de_b} = \frac{\phi_a (D,A) (\phi_b(D,A) - s_b)}{\phi_b(D,A)\phi_a (A,A) - (\phi_b(D,A) - s_b) \phi_a (D,A)} > 0$$

as long as $e_a^{(D,D)} > 0$. This is quite intuitive as an increase in e_b increases the effort necessary to prevent attack from the contestant as everything else given, an increase in e_b, increases the contestant's probability of winning.

It is also quite intuitive to understand that an increase in the peace share of the contestant is going to make it easier to implement peace using effort.

$$\frac{de_a^{(D,D)}}{ds_b} = \frac{-e_b \phi_a (D,A) \phi_b(D,A)\phi_a (A,A)}{(\phi_b(D,A)\phi_a (A,A) - (\phi_b(D,A) - s_b) \phi_a (D,A))^2} < 0.$$

It is interesting to observe that, even it will not always be possible to implement (D,D) using effort. The reason is that also effort has a direct negative impact on the contestant's probability of winning, it also has an indirect positive effect through its impact in the degree of differentiation that would be chosen in (D,A).

For effort to make contestants prefer no conflict we need

$$s_b \geq \phi_b (D,A) \frac{t_3 + e_b}{t_3 + e_b + e_a},$$

which can be rewritten as

$$s_b \geq \phi_b (D,A) \left[1 - \frac{e_a \phi_a (A,A)}{(e_a + e_b) \phi_a (D,A)} \right].$$

For the above to happen under $s_b < \phi_b (D,A)$, we need

$$\phi_b(D,A) \frac{\phi_a (D,A) - \phi_a (A,A)}{\phi_a (D,A)} < s_b,$$

this condition is also the one that ensures a positive $e_a^{(D,D)}$.

Note that, for instance, if $\phi_a (A,A)$ very small, the sign of the above might be positive, meaning that an increase in effort would encourage stronger increases differentiation as, it is less difficult to block (A,A), making (D,A) relatively more attractive.

A smaller $\phi_b(D,A)$, a higher s_b or a smaller rate of difference between $\phi_a(D,A)$ and $\phi_a(A,A)$ will all decrease the contestants incentive to choose (D,A) over (D,D) and therefore, it will be more likely that (D,D) becomes feasible.

Also note that

$$e_a^{(D,D,t=0)} = \frac{e_b\left(\phi_b(D,A)-s_b\right)}{s_b} < e_a^{(D,D)} = \frac{e_b\phi_a(D,A)\left(\phi_b(D,A)-s_b\right)}{\phi_b(D,A)\phi_a(A,A)-\left(\phi_b(D,A)-s_b\right)\phi_a(D,A)}$$

iff

$$\phi_b(D,A)\left(\phi_a(A,A)-\phi_a(D,A)\right)<0.$$

The above means that if under technological differentiation, (D,D) can be implemented using effort, then this level of effort will be bigger than the level of effort required to implement (D,D) in the absence of technological differentiation. Intuitively this is due to the fact that the contestant has a higher incentive to shift to (D,A) when it can differentiate.

3.3.2 Incumbent's effort that implements (D,A) as the unique Pure Strategy Nash Equilibrium in the last stage of the game

We denote the level of effort that the incumbent would do if it wanted to target (D,A) as $e_a^{(D,A)}$. This effort can be obtained from the following maximization problem

$$\underset{\{e_a\}}{Max} \qquad EU_a(D,A)=\phi_a(D,A)\,P_a(D,A)\,V-C(e_a),$$

substituting for $P_a(D,A)$ we get

$$\underset{\{e_a\}}{Max} \qquad EU_a=\phi_a(D,A)\frac{\Phi+e_a}{\Phi+t_3(e_a)+e_b+e_a}V-C(e_a).$$

We then obtain the First Order Condition:

$$\begin{aligned}\frac{dEU_a}{de_a} &= \phi_a(D,A)\,V\left(\frac{t_3(e_a)+e_b}{\left(\Phi+t_3(e_a)+e_b+e_a\right)^2}+\frac{-\left(\Phi+e_a\right)\frac{dt_3(e_a)}{de_a}}{\left(\Phi+t_3(e_a)+e_b+e_a\right)^2}\right)- \\ -\frac{dC(e_a)}{de_a} &= 0.\end{aligned}$$

The above tells us that as long as $\frac{dt_3(e_a)}{de_a}>0$, the fact that the incumbent is forward looking reduces the optimal amount of effort when (D,A) is the equilibrium targeted.

Substituting for $t_3(e_a)$ in the expected utility equation, we get:

$$EU_a(D,A) = \frac{\phi_a(A,A)\, e_a}{(e_a + e_b)} V - C(e_a).$$

The above is equivalent to the expected incumbent's utility if (A, A) was the expected equilibrium. This is due to the fact that the incumbent advances that in the following stage the contestant will chose the degree of technological differentiation that just makes the incumbent prefer (D, A) to (A, A).

After substituting for all derivatives the first order condition simplifies to:

$$\frac{dEU_a(D,A)}{de_a} = V \frac{\phi_a(A,A)\, e_b}{(e_a + e_b)^2} - \frac{dC(e_a)}{de_a} = 0.$$

Proposition 2. *If the incumbent targets (D, A), when the contestant has the option of technologically differentiating from incumbent, the outcome is a lower level of effort by the incumbent as long as "entrenching" advantages, represented by Φ are sufficiently small and $\phi_a(D,A) > \phi_a(A,A)$.*

<u>Proof</u>

In the case where there technological differentiation is not a possibility, the equivalent first order condition for incumbent effort would be:

$$\frac{dEU_a(D,A)}{de_a} = V \frac{\phi_a(D,A)\, e_b}{(\Phi + e_b + e_a)^2} - \frac{dC(e_a)}{de_a} = 0.$$

Therefore a condition for e_a to be lower than in the absence of technological differentiation

$$\frac{\phi_a(A,A)}{(e_a + e_b)^2} < \frac{\phi_a(D,A)}{(\Phi + e_b + e_a)^2}.$$

Note that if $\Phi = 0$, the above condition would be equivalent to $\phi_a(D,A) > \phi_a(A,A)$. However, if $\phi_a(A,A) = \phi_a(D,A)$, the inequality will clearly reverse, in such case, the level of effort by the incumbent, without technological differentiation by the contestant, would actually be lower$\square$

Finally, we calculate the equilibrium $e_a^{(D,A)}$ for $\frac{dC(e_a)}{de_a} = c$ (it can be checked that $EU_a(D,A)$ is concave even with constant marginal costs), substituting in the first order condition we obtain:

$$e_a^{(D,A)} = \sqrt{e_b}\left[\sqrt{\frac{V\phi_a\left(A,A\right)}{c}} - \sqrt{e_b}\right].$$

Note that:

$$\frac{de_a^{(D,A)}}{dV} > 0.$$

$$\frac{de_a^{(D,A)}}{dc} < 0.$$

$$\frac{de_a^{(D,A)}}{de_b} = \frac{\sqrt{V\frac{\phi_a\left(A,A\right)}{c}}}{2\sqrt{e_b}} - 1 > 0 \text{ iff } \sqrt{\frac{V\phi_a\left(A,A\right)}{c}} > 2\sqrt{e_b}.$$

An increase in V increases the incentive to put effort in order to increase probability of winning in a (D,A) type of conflict. An increase in marginal cost of effort naturally decreases incentive to put effort though. Impact of e_b is ambiguous.

3.3.3 The decision of the incumbent on whether to induce (D,D) or (D,A)

The incumbent will induce no conflict, (D,D), iff $EU_a\left(e_a^{(D,D)}\right) > EU_a\left(e_a^{(D,A)}\right)$.

We have three main cases under the assumption that $s_a > \phi_a\left(D,A\right)$ (this assumption important here, it effectively excludes (A,D) as a possible equilibrium without having to analyze the contestant's blocking of such equilibrium, this will help the comparison of technological differentiation and no differentiation case analyzed next section and it will ensure that all equilibria presented will be unique, therefore, however, note also that (A,A) was in principle a possible equilibrium but the contestant blocks it by choosing t_3, which just falls short of making incumbent prefer it to (D,A)):

1. (D,D) feasible for any level of the incumbent's effort. This is the simplest case. The incumbent will then choose $e_a^{(D,D)} = 0$ and there will not be technological differentiation. This case will arise when $\phi_b(D,A) < s_b$.

2. $e_a^{(D,A)} > e_a^{(D,D)}$. Given our assumption that for given effort, the incumbent prefers no war to (D,A), $e_a^{(D,D)}$will be implemented and technological differentiation will be zero (if there was any associated cost to it), although even if it was positive incumbent wouldn't care as an attack will not occur.

3. $e_a^{(D,A)} < e_a^{(D,D)}$. Note that this implies that $t_3\left(e_a^{(D,A)}\right) > t_1\left(e_a^{(D,A)}\right)$ since otherwise, by definition of $e_a^{(D,D)}$(lowest level of effort that ensures $t_3 = t_1$), we would have a contradiction. In this case, if

$$EU_a\left(e_a^{(D,A)}\right) > EU_a\left(e_a^{(D,D)}\right) \Leftrightarrow$$
$$\Leftrightarrow \phi_a(D,A)\frac{\Phi + e_a^{(D,A)}}{\Phi + t_3(e_a^{(D,A)}) + e_b + e_a^{(D,A)}}V - C\left(e_a^{(D,A)}\right) > s_a V - C\left(e_a^{(D,D)}\right).$$

then (D,A) will be the equilibrium. Note that the above condition requires $e_a^{(D,A)} < e_a^{(D,D)}$.

Based on our comparisons between the benchmark (no differentiation possible) case and our model with differentiation the following result becomes apparent

Proposition 3. *A decision by the incumbent to induce* (D,D)*, that is, to prevent conflict, will become less feasible and less likely to be adopted by the incumbent, even if feasible, if differentiation by the contestant is a possibility.*

Simply recall that it takes more effort to persuade the contestant not to attack when the can technologically differentiate, which gives them an advantage in (D,A) situation, $e_a^{(D,D,t=0)} < e_a^{(D,D)}$. Besides, when the contestant can technologically differentiate it actually decreases the incentive of incumbent to put effort down to just above the incentive to put effort in a (A,A) type of conflict and therefore, $e_a^{(D,A)} < e_a^{(D,A,t=0)}$. This makes it less likely that $e_a^{(D,D)} < e_a^{(D,A)}$.

4 Conclusion

In this paper, we have presented a simple model that captures some of the issues and difficulties involved in managing asymmetric conflict.

We have proved that the possibility of differentiation from an incumbent power's conflict technology by a contestant group makes it harder for the incumbent to want to implement peace by means of effort. In some cases, it may make it altogether unfeasible. The incumbent power is then more likely to adopt a defensive strategy to cope with a defensive type of conflict. This strategy actually involves less effort that the one that would be made if a defensive strategy was chosen to be induced in the absence of possible differentiation by the incumbent.

We have also proved that, when a defensive conflict is expected, increases in the incumbent's effort simply encourage technological differentiation of the side of the contestant power.

Our model has potential applications in the industrial organization literature. It becomes a bit more difficult to see how the final stage of the game may relate to the issues refer to in the computing industry, it is still possible to see a link though. We could think of the incumbent as an established firm with a deep pocket. The contestant may then be seen as a new entrant which may choose indirect entry with a product horizontally different from the incumbent's, targeting a consumer base with a potential for future growth. The actual conflict can the be seen as an advertising war, which may be a damaging mutually aggressive advertising campaign in which both firms are trying to persuade consumers that the other product is not worth buying or defensive, in which one of two firms just try to defend the status quo market share.

The model presented in the paper does not consider the introduction of incomplete information for any of the variables. A number of papers within the terrorism literature have used models of incomplete information. Lapan and Sandler (1993) and Overgaard (1994) present an attack by a terrorist group as a signal of the terrorist effort. The introduction of such type of asymmetric information in our model could be an interesting future line of research.

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