

## Growth and Productivity

- Long run economic growth is obviously rather important and economics should have a lot to say about it

### There are five stylized facts

1. Large differences in growth rates and levels of per capita income persist across countries and over time
2. In the long run economies exhibit a balanced growth path -Q per capita and K per capita roughly same positive growth over time
3. real rate of return to capital roughly constant long run and real wages grow at rate close to growth Q per head. So K/Q no trend over time and K/Q and K/Q stay roughly constant
4. High investment rates in capital and human capital (education) closely related to high living standards
5. Low population associated with high living standard

### Solow Model

- Concept of capital
  - productive but not all the time and included inventories
  - created from existing resources -trade off cons/invest
  - earns a return or is rented
  - depreciates
  - is a rival good
- Concept of aggregate capital stock
  - capital controversy
- Constant returns to scale production function and diminishing MP for all factors

$$Y = f(K, L)$$

- Convenient to define in intensive form

$$y = \frac{Y}{L} = \frac{1}{L}F(K, L) = F\left(\frac{K}{L}, 1\right) = f(k)$$

- CRS also means average and marginal returns to a factor depend only on the factor ratio

- Diminishing returns means APK and MPK are decreasing functions and APL and MPL increasing functions of  $\frac{K}{L}$
- To get unique steady state need further assumptions, but in practice often work with form that already has them, such as the C-D production function

$$Y = K^\alpha L^{1-\alpha}$$

or

$$y = k^\alpha$$

- Assume labour force grows at constant  $n$  ie  $L_t = L_0 \exp(nt)$
- Assume exogenous savings rate  $s$  and  $\dot{K} = I - \partial K$  with  $\partial$  the rate of depreciation we can that the growth of the capita stock will depend on the APK and is a declining function of the capital labour ratio and if the capital stock grwos at the same rate as the constant labour force growth, the capital labour ration will be constant ie at steady state  $k^*$
- We can also derive the Fundamental Solow Equation:

$$\dot{k} = sf(k) - (\partial + n)k$$

- the first term is the amount investment is adding to the capital stock per worker
- the second is the amount of capital required to offset depreciation and to equip additions to the labour force at existing levels of capital stock per head.
- Can rewrite by dividng by k

$$g_k = \frac{sf(k)}{k} - (\partial + n)$$

- The steady state
 
$$y^* = f(k^*)$$
- will occur at the level of effective capital where these are equal
- Can also show that it implies growth output is weighted average of the growth of inputs and the weights are the factor shares
- But rather unsatisfactory as the model predicts steady state of constant level of output per worker, but a clear stylised fact is that per capita output has been increasing.
- So need to consider technology in model

### Introducing technology

- Harrod neutral technological progress -disembodied labour augmenting. So effect of TP on output is through labour, increasing productivity for a given capital stock.

$$Y = F(K, A_t L)$$

- Alternatives:

$$\text{Hicks neutral -factor augmenting} \quad Y = A_t F(K, L)$$

$$\text{Solow neutral -capital augmenting} \quad Y = F(A_t K, L)$$

- For C-D all 3 are identical, but otherwise only harrod neutral defines steady state with positive per capita growth
- Technology is essentially 'manna from heaven'
- Use labour measured in efficiency units

$$\hat{y} = \frac{Y}{AL} = \frac{1}{AL} F(K, L) = F(\hat{k}, 1) = f(\hat{k})$$

- Gives new version of Fundamental Solow equation

$$g_{\hat{k}} = \frac{s f(\hat{k})}{\hat{k}} - (\delta + \chi + n)$$

- with  $\chi$  the exogenous rate of TP
- Net investment in physical capital takes place when savings outweigh the capital
  - required to replace depreciation,
  - required to employ additional population
  - made profitable by increased productivity of existing workforce due to TP
- Gives result that the output per worker and capital grow at the exogenous rate of TP

### Growth accounting: Dennison/Maddison following Solow

- Solow had tried to account for the contribution of factors of production and so any unaccounted for growth was technological progress.

$$F(K, L, A)$$

$$\implies \dot{Y} = F_K \dot{K} + F_L \dot{L} + F_A \dot{A}$$

$$\implies g_y = \frac{F_K K}{Y} g_k + \frac{F_L L}{Y} g_L + \frac{F_A A}{Y} g_A$$

- Assuming competitive markets so factors paid their MPs and call the last term the Solow residual

$$\implies g_y = \frac{rK}{Y} g_k + \frac{wL}{Y} g_L + SR$$

- so

Output growth =  $a$  capitalgrowth +  $b$  labourgrowth + a residual

with  $a$  being the share of profits and  $b$  the share of wages

- So source of growth are simply contributions from factor inputs and Total Factor Productivity (the residual)
- Even when extended studies have found the residual does a lot of work.
- Growth without technological progress suggests that output and employment grow at same level, i.e. labour productivity does not grow,
- But this is not what is observed so need to consider what drives growth productivity and divergent growth rates across countries

### Convergence/Catch up:

- Solow developed model to look at dynamics of individual country, but used for cross country studies.
- Implication of model with TP is that countries with low levels of capital will grow more quickly than those with high, so expect poorer countries to catch up with richer ones over time.
- Actually poorer will need to increase savings rate (lower popn growth) relative to rich
- this is the hypothesis of absolute/unconditional convergence
- Empirical analysis suggest it doesnt hold for all countries, but may do for subsets with similar characteristics
- Also have conditional convergence, which depends on being away from steady state, if steady state characteristics are similar. Poorer countries will only grow faster if are further away from their steady state than the richer ones.
- Can catch up through education; through relaxing assumption of universal access to technology so countries catch up by adopting frontier technologies if education sufficient.

- Faster catch-up further behind countries are
- Why expected former communist countries to grow fast -underestimated importance of social relations
- Some evidence of convergence if look at advanced economies
  - but also examples of developing countries that have failed to catch up
  - also Friedmans criticism/fallacy-Argentina in early part of century and now and sample selection bias
  - Still considerable debate -use of panel data

**Human capital:** can broaden concept of capital to include human capital and this fits empirical reality better.

Introduce it into the production function.

Using the C-D

$$Y = K^\alpha H^\beta (AL)^{1-\alpha-\beta}$$

we get

$$y^* = A_t \left( \frac{s_k}{\partial + \chi + n} \right)^{\frac{\alpha}{1-\alpha-\beta}} \left( \frac{s_h}{\partial + \chi + n} \right)^{\frac{\beta}{1-\alpha-\beta}}$$

so rich countries have high savings rates and low population growth and access to advances in technology

- Mankiw, Romer & Weil (1992) QJE estimate this form of model for a sample of 98 countries

$$\begin{aligned} \log y_t - \log y_0 &= a - b \log y_0 + b \frac{\alpha}{1-\alpha-\beta} \log s_k + b \frac{\beta}{1-\alpha-\beta} \log s_h \\ &\quad + b \frac{\alpha + \beta}{1-\alpha-\beta} \log(n + \chi + \partial) + \epsilon \end{aligned}$$

- find it is an improvement, they also consider the convergence rate with human capital and find strong evidence for conditional convergence
- Other extensions of Solow Swan Model -see Carlin and Soskice
  - Welfare -consumption and the 'Golden Rule'
  - Endogenous Savings -Ramsay Model
  - Government and taxation